Edukasiana: Jurnal Inovasi Pendidikan
ISSN 2809-6770
Volume 4, Issue 3, 2025, pp. 1337 – 1350
DOI: https://doi.org/10.56916/ejip.v4i3.2358

Exploring Students' Critical Thinking Skills Through an Analysis of Their Responses to Trigonometry Problems

Dadan Ramadhan*, Mohamad Gilar Jatisunda[2]
[1]Department of Mathematics Education, Universitas Majalengka, Majalengka, Indonesia
[2]Department of Mathematics Education, Universitas Majalengka, Majalengka, Indonesia
*Corresponding Author: ddan040104@gmail.com

Article History:
Received 2025-06-27
Accepted 2025-07-30

Keywords:
Critical thinking
Trigonometry
Student errors
NVivo
Mathematics learning

ABSTRACT

This study aims to analyze students' critical thinking skills in solving trigonometry problems using a qualitative case study approach. The participants were 29 twelfth-grade students from SMAN 1 Kadipaten. Data were collected through classroom observations, students' written responses, and semi-structured interviews, and were analyzed using NVivo 14 software. The findings revealed that only a small proportion of students demonstrated adequate critical thinking indicators, particularly in the areas of interpretation, analysis, inference, and evaluation. While 69% of students were able to construct appropriate mathematical models, only 31% correctly applied relevant formulas, and merely 3% succeeded in accurately stating the final answers. The interview results supported the analysis, indicating that many students faced conceptual misunderstandings, procedural difficulties, and lacked reflective thinking in validating their solutions. These findings underscore the need to enhance students' critical thinking skills through visual learning tools, reflective discussions, and contextual problem-solving exercises. This study contributes to the development of diagnostic assessment instruments and the design of adaptive instructional strategies in mathematics education, particularly for complex topics such as trigonometry.

1. INTRODUCTION

Critical thinking is recognized as an essential cognitive competency that students must possess to meet the demands of 21st-century learning (Halim, 2022). In mathematics education, it extends beyond understanding concepts to evaluating open-ended situations, identifying relationships, tracing cause–and–effect patterns, drawing logical conclusions, and analyzing relevant data (Siahaan & Meilani, 2019; Jablonka, 2014). Its development is a priority because it fosters logical reasoning, complex problem-solving, and rational decision-making (Utami, 2022; Afriansyah et al., 2021). Furthermore, critical thinking forms the basis of higher-order mathematical reasoning, enabling learners to apply mathematical concepts in real-life contexts (Syafril et al., 2020; Sachdeva & Eggen, 2021). Despite its acknowledged importance, the integration of critical thinking into classroom practice remains insufficient, leading to gaps between curriculum expectations and students' actual competencies. Addressing this issue is crucial for preparing students to engage critically and meaningfully with mathematical problems.

Several studies reveal that students' critical thinking ability remains low. Pakpahan et al. (2023) reported that students face difficulties in meeting the indicators of interpretation, analysis, evaluation, and inference. Fitriani and Kowiyah (2022) similarly found that more than half of students failed to meet these indicators in mathematical problem-solving. Such limitations hinder students' ability to communicate ideas, reason logically, and connect mathematical concepts meaningfully (Pangestika & Faiziyah, 2022). Obstacles often stem from limited learning activities that encourage idea exploration and reflective dialogue (Özkaya & Semerci, 2022; Cáceres et al., 2020). Furthermore, gaps between 21st-century curriculum demands and classroom implementation contribute to poor performance (Franklin et al., 2022; Ongesa, 2020). Assessment tools are also often inadequate for authentically measuring critical thinking (Braun et al., 2020; Shavelson et al., 2019). This misalignment between importance and practice underscores the urgency of addressing critical thinking as a pressing educational concern (Dwyer, 2023; Firdaus et al., 2015).

One potential approach to improving critical thinking in solving trigonometry problems is through detailed analysis of students' answers to identify weaknesses and latent potential. Such analysis serves as both an evaluative and diagnostic tool, offering insights into thought processes, reasoning patterns, and problem-solving strategies (Cahya & Juandi, 2021; Kükey & Aslaner, 2023). This is particularly relevant for trigonometry, a subject known for its complexity due to the integration of spatial, symbolic, and conceptual understanding. Previous studies indicate that students struggle with basic trigonometric concepts, often making errors in formula application and graph interpretation (Nurmeidina & Rafidiyah, 2019; Nanmumpuni & Retnawati, 2021; Hidayati, 2020; Obeng et al., 2024). Both explicit and implicit misconceptions exacerbate these challenges, limiting logical problem-solving in trigonometry (Hamzah et al., 2021; Malambo, 2021; Ahmad et al., 2018). Addressing these difficulties requires analytical tools that are both reliable and systematic in uncovering the depth of students' reasoning.

In this study, NVivo software is employed as a qualitative analysis tool to examine students' responses to trigonometry problems. NVivo has proven effective for managing, coding, and visually representing textual data in a thematic manner, consistent with principles of discourse-based textual analysis (Lopezosa, 2020; Müller de Andrade et al., 2020). This research aims to analyze the characteristics of students' critical thinking in solving trigonometry problems through a qualitative, answer-based approach. It seeks to identify critical thinking indicators present in student responses and recurring error patterns. The study's findings are expected to contribute theoretically to the development of diagnostic instruments for assessing critical thinking and practically to guiding teachers in designing evidence-based interventions and evaluations. For operational purposes, critical thinking here refers to students' ability to meet the indicators of interpretation, analysis, evaluation, inference, and logical explanation in trigonometric problem-solving.

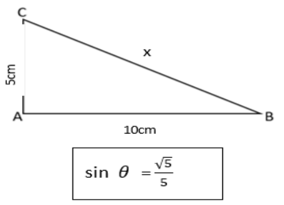
2. METHODOLOGY

This study employed a qualitative approach with an intrinsic case study design to explore students' critical thinking abilities in mathematics learning, specifically in the context of trigonometry. The qualitative approach was selected because it enables an in-depth understanding of complex phenomena within natural settings and focuses on the meanings constructed by participants (Creswell & Poth, 2018; Yin, 2018). The intrinsic case study design was considered appropriate, as the research aimed to examine the phenomenon as a unique case rather than for generalization purposes. This methodological alignment ensured that the data collection and analysis processes were coherent with the research objectives.

The research was conducted during the even semester of the 2024/2025 academic year at SMAN 1 Kadipaten, Majalengka, Indonesia. The study population consisted of all twelfth-grade students, from which a purposive sample of 29 students was selected as the primary participants. The selection was based on the inclusion criteria of having completed prerequisite mathematics topics, while exclusion criteria included prolonged absence or inability to participate in all research activities. For the interview phase, eight students were purposively selected to represent varying levels of performance, ensuring diverse perspectives and experiences.

The main instruments consisted of trigonometry problem sets, an observation guide, and a semi-structured interview protocol. The trigonometry problems were designed to elicit indicators of critical thinking based on Facione's (2011) framework, including interpretation, analysis, evaluation, inference, explanation, and self-regulation. Table 1 presents the problem items, Newman's error analysis aspects, and the associated critical thinking indicators. The observation guide was used to document classroom interactions and student behaviors, while the interview protocol was developed to probe students' reasoning processes, difficulties, and strategies. All instruments underwent expert validation through review by two mathematics education lecturers and one experienced mathematics teacher.

Table 1. Trigonometry Problem Instruments

No	Problem	Newman's Aspects	Critical Thinking Indicators
1	In a right triangle, there are six basic trigonometric functions: sine, cosine, tangent, cosecant, secant, and cotangent. Explain each function and write its formula based on the sides of a right triangle.	Reading & understanding, transformation, process skills, and answer writing.	Explain the definitions and functions of sine, cosine, and tangent in right triangles.
2	Given a right triangle ABC where the hypotenuse AC = 7√2 cm and AB = 6 cm, determine BC.	Reading & understanding, transformation, process skills, and answer writing.	Determine missing side lengths using the Pythagorean theorem.
3	A task requires finding cos 30° from various sources. One source states cos 30° = 0.154251, another states 123. Compare results, determine which is more accurate, and explain your reasoning.	Reading & understanding, transformation, process skills, and answer writing	Compare results from multiple sources and assess their accuracy.
4	 From the given information, determine whether sin θ = √5/5 can be calculated. If so, verify the value; if not, identify missing information. Then determine cos θ.	Reading & understanding, transformation, process skills, and answer writing	Identify necessary assumptions for trigonometric ratio calculations.
5	An airplane takes off at a 30° elevation angle and travels 500 m diagonally. Determine its horizontal distance from the runway.	Reading & understanding, transformation, process skills, and answer writing	Plan an effective calculation sequence for a real-world trigonometric application.

Data collection involved three main stages. First, a classroom observation was conducted during a mathematics lesson on trigonometry to capture the natural learning process. Second, students completed the trigonometry problem set under controlled classroom conditions, with their written responses collected for analysis. Third, semi-structured interviews were conducted with the eight selected students to explore their reasoning, challenges, and strategies in solving trigonometry problems (Halim, 2022). Interviews provided access to students' personal narratives and mental representations, yielding rich, contextual, qualitative data (Kvale & Brinkmann, 2009; Merriam & Tisdell, 2016).

The primary construct examined was critical thinking ability, operationally defined as the extent to which students fulfilled the indicators of interpretation, analysis, evaluation, inference, explanation, and self-regulation (Facione, 2011) in solving trigonometry problems. These indicators were embedded in both the problem set and the interview questions to ensure consistency and construct validity. Thematic analysis was applied following the steps of data reduction, data display, and conclusion drawing, as outlined by Miles et al. (2014). NVivo 14 software was utilized to manage, code, and extract relevant themes from textual data, ensuring a systematic and transparent analytic process. The use of NVivo is supported by prior research demonstrating its effectiveness in discourse-based and education-related qualitative studies (Woods et al., 2016; Müller de Andrade et al., 2020). Coding was conducted both deductively, based on Facione's indicators, and inductively to capture emerging themes beyond the predefined framework.

This study adhered to established research ethics protocols. Informed consent was obtained from all participants, with assurances of confidentiality and anonymity. Participation was voluntary, and students were free to withdraw at any time without incurring academic penalties. Ethics approval was secured from the relevant institutional review board prior to data collection. The research was conducted over a period of three months, comprising one month of instrument development and validation, one month of data collection, and one month of data analysis and reporting.

3. RESULTS AND DISCUSSION

Results

After collecting data through observation and semi-structured interviews, the researcher proceeded with data analysis using an open coding approach, assisted by NVivo 14. Segments of student behavior relevant to the indicators of critical thinking skills were coded and organized into five main themes: understanding and recording information, analyzing the problem, developing and applying the mathematical model, applying the formula, and writing the final answer. Each theme was further divided into two performance categories—"able" and "unable." This thematic categorization allowed the researcher to map the extent to which students demonstrated specific indicators of critical thinking when solving trigonometry problems.

For example, in the understanding and recording information stage, students classified as *"able"* demonstrated the capacity to accurately identify both the known and unknown elements of a trigonometry problem, distinguishing between the data provided and the information required to reach a solution. These students demonstrated careful reading, attention to detail, and the ability to translate verbal descriptions into structured notes or diagrams, thereby creating a clear foundation for subsequent analytical steps. In contrast, students categorized as *"unable"* frequently misunderstood the problem statement, misinterpreted critical details, or overlooked essential information entirely. Such gaps in comprehension often led to confusion in later stages, as they began problem-solving with an incomplete or inaccurate understanding of the task. This stage is foundational to mathematical reasoning, as accurate comprehension not only shapes the selection of strategies but also reduces the likelihood of procedural errors, thereby enhancing the overall effectiveness and accuracy of the problem-solving process.

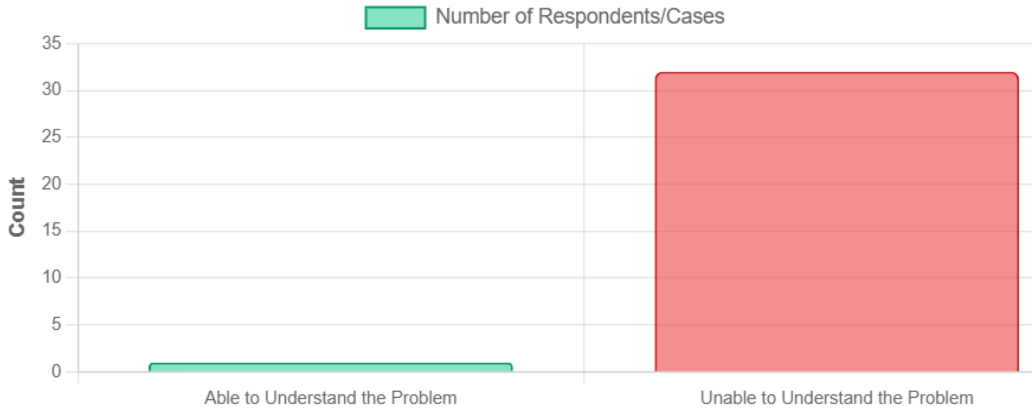


Figure 1. Understanding and Writing Down Information

Analysis revealed that out of 29 students, only one could correctly understand and record what was known and what was asked in the problem. The remaining 28 struggled at this stage, showing weaknesses in basic comprehension skills when reading and interpreting mathematical information. Many failed to distinguish between the given data and the desired solution. Without this skill, students are likely to face further difficulties in the following stages of critical thinking. This pattern continued into the reading and problem-solving stages.

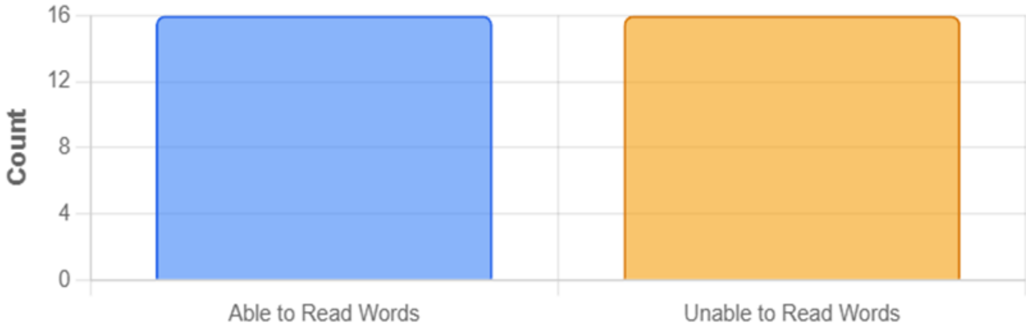


Figure 2. Reading the Problem

Here, 14 students (48%) demonstrated adequate ability to read and comprehend trigonometry problems, identifying essential information, grasping the question's intent, and relating data to relevant trigonometric concepts. In contrast, 15 students (52%) experienced difficulties, including extracting key information, recognizing mathematical keywords, and understanding the logical structure of contextual problems. This indicates that a substantial portion of the class lacks sufficient mathematical literacy, which can hinder both conceptual understanding and the ability to reason critically. Progressing to the determining and writing the mathematical model stage,

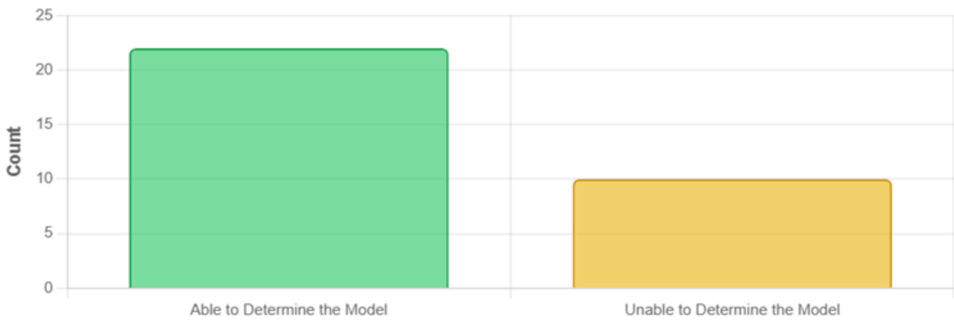


Figure 3. Determining and Writing the Mathematical Model

20 students (69%) were able to transform verbal descriptions into accurate mathematical models. This demonstrated their analytical thinking in recognizing relationships between variables, translating keywords into symbols, and formulating equations. However, nine students (31%) faced errors in selecting appropriate trigonometric concepts, relating triangle elements, or converting verbal statements into symbolic form—missteps that can disrupt the entire problem-solving process. These challenges became more evident during the application of the formula stage.

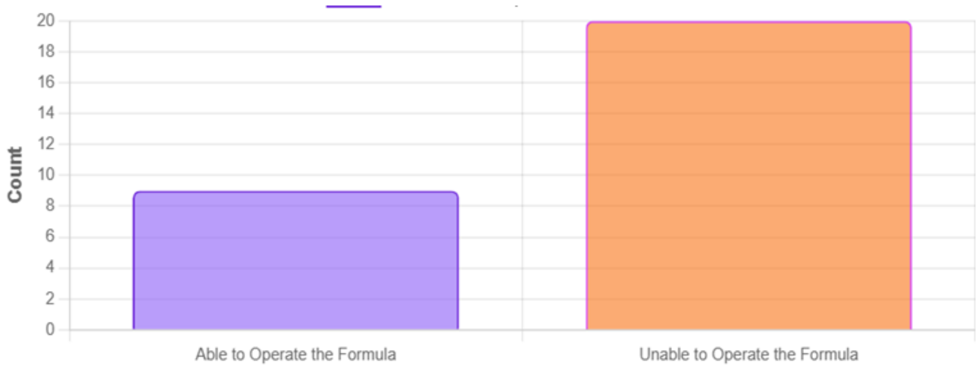


Figure 4. Applying the Formula

Only nine students (31%) demonstrated the ability to correctly select and apply the appropriate trigonometric formulas while performing calculations with a high degree of accuracy. In contrast, the majority of students (69%) encountered significant difficulties, including the incorrect selection of formulas, improper substitution of known values, and frequent computational errors. These challenges underscore a pronounced gap between representational understanding, the ability to construct an accurate mathematical model of the problem, and procedural competence, which involves executing the necessary operations systematically and accurately. This gap represents a common and critical barrier in mathematical problem-solving, as even a sound conceptual model will fail to produce a correct solution if procedural execution is flawed. Furthermore, the subsequent and final stage—writing the final answer—proved to be the most demanding for students. At this stage, errors in prior steps, combined with inadequate verification and reflection, significantly reduced the accuracy and contextual appropriateness of the solutions produced.

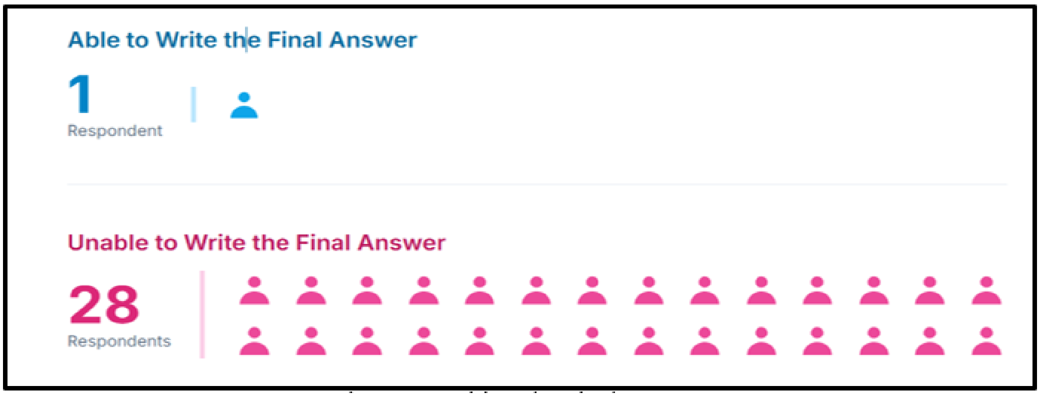


Figure 5. Writing the Final Answer

At this final stage of problem-solving, only one student (3%) was able to produce a solution that was complete, mathematically correct, and contextually appropriate. The overwhelming majority failed to meet these criteria, mainly because they did not conduct a thorough verification of their work or neglected to consider the significance of units and contextual relevance in their answers. These omissions suggest that many students lack systematic self-checking habits, which are essential for identifying and correcting errors before final submission. Additionally, limited reflection skills—such as reviewing the logical flow of their solution, cross-checking results with the problem statement, or assessing the plausibility of their answers—contributed to the low overall quality of final responses. Recognizing these patterns, the researcher turned to qualitative interview data to explore the underlying cognitive and behavioral factors. This deeper analysis aimed to uncover how students' perceptions, learning habits, and problem-solving strategies influenced their performance in this final and crucial stage.

Figure 6. Word Cloud from Interview Results

The NVivo-generated word cloud analysis revealed a concentration of recurring keywords, including *"teaching aids," "visual," "difficult," "confused,"* and *"mathematics."* The frequent appearance of these terms suggests that a substantial number of students struggle to comprehend trigonometric concepts when instruction relies solely on abstract explanations. Instead, they appear to benefit more from the integration of visual representations, concrete materials, and interactive media that bridge symbolic mathematics with tangible experiences. The prominence of the term *"confused"* reflects students' cognitive struggle in connecting procedural steps with underlying conceptual frameworks, while *"difficult"* underscores the perceived complexity of trigonometric topics. Notably, the emergence of the term *"enjoyable"* suggests that when visual and interactive methods are employed, students display greater engagement, motivation, and receptivity to learning. These qualitative insights complement the quantitative cluster analysis, jointly highlighting the critical role of multimodal instructional strategies in addressing both conceptual and procedural barriers to mathematical mastery.

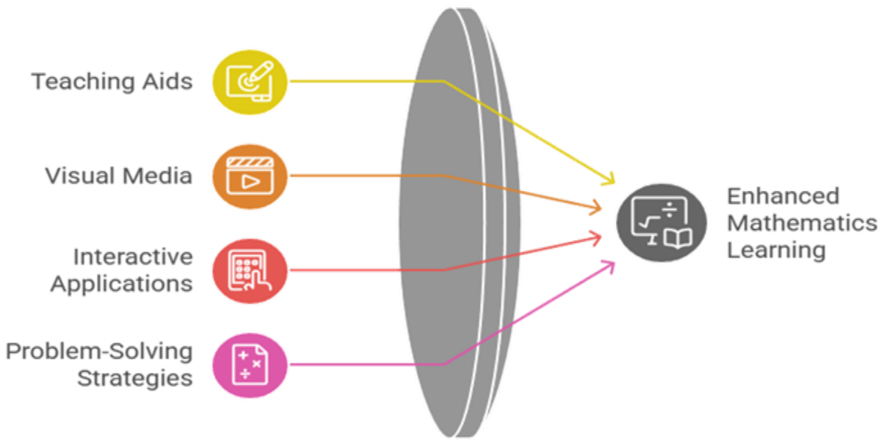


Figure 7. Pathways to Mathematical Mastery

The analysis identified two interrelated categories of difficulty that students face when solving trigonometric problems: conceptual difficulties and procedural difficulties. Conceptual difficulties encompass challenges in understanding the fundamental definitions of trigonometric ratios, recognizing the properties and relationships of trigonometric functions, and connecting these concepts to real-world contexts or geometric representations. Students with weak conceptual foundations often struggle to accurately interpret problem statements or identify the most suitable mathematical approach. Procedural difficulties, on the other hand, involve errors in selecting the correct formula, organizing solution steps logically, and performing accurate numerical calculations. These procedural gaps frequently stem from insufficient practice, limited familiarity with problem-solving strategies, or a lack of systematic verification at each step. Importantly, the two categories are not independent; conceptual misunderstandings can lead to procedural mistakes, while repeated procedural errors may deepen conceptual confusion. This bidirectional relationship underscores the need for instructional interventions that simultaneously enhance conceptual comprehension and procedural fluency.

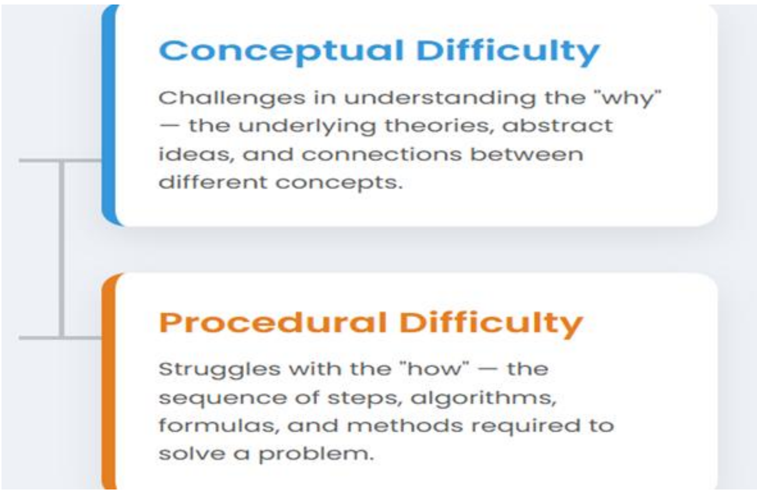


Figure 8. Cluster Diagram of Difficulties

This diagram illustrates the dynamic and reciprocal relationship between conceptual challenges and procedural challenges in students' trigonometric problem-solving processes. A weak conceptual understanding—such as limited knowledge of trigonometric definitions, poor grasp of function properties, or inability to relate concepts to real-life contexts—often results in procedural errors. For example, students who fail to internalize the meaning of sine, cosine, and tangent may incorrectly select formulas or misinterpret the relationships among triangle elements when applying them in problem contexts. Conversely, repeated procedural mistakes—such as incorrect formula substitution, disorganized solution steps, or persistent computational inaccuracies—can further erode conceptual clarity. These repeated procedural failures not only hinder immediate problem-solving success but also foster misconceptions, reduce students' confidence, and weaken their motivation to engage with the subject matter. Over time, this cyclical interaction between conceptual and procedural weaknesses forms a self-reinforcing barrier, making it increasingly difficult for students to develop higher-order mathematical thinking and critical reasoning skills.Recognizing this interplay is therefore essential for educators and curriculum designers when developing targeted instructional interventions. Effective strategies should address both dimensions concurrently—for instance, integrating conceptual scaffolding (explicit concept explanations, visual models, and analogies) with procedural practice (step-by-step guided problem-solving, error analysis, and immediate feedback). By breaking the cycle of mutual reinforcement between conceptual and procedural difficulties, instructional designs can help students build a more stable and interconnected knowledge structure, ultimately leading to greater accuracy, efficiency, and confidence in solving trigonometric problems.

Discussion

This study investigates students' critical thinking skills in solving trigonometry problems, revealing a substantial gap between curriculum expectations and actual competence. This finding aligns with Tran and O'Connor (2024), who emphasize that teachers' curriculum competence plays a pivotal role in achieving intended learning outcomes. The analysis indicates that most students struggled to meet Facione's critical thinking indicators, interpretation, analysis, inference, and evaluation, consistent with Sachdeva and Eggen's (2021) observation that such difficulties are often linked to limited metacognitive skills in mathematics learning. Conceptual challenges specific to trigonometry have also been documented by Nanmumpuni and Retnawati (2021), while Angraini and Masykur (2018) highlight the value of problem-based learning (PBL) modules in addressing these barriers. Supporting this, Maulidiya and Nurlaelah (2019) demonstrate that PBL can significantly enhance mathematical critical thinking skills, making it a promising approach for narrowing the gap between curricular goals and students' actual performance.

The results further reveal a sharp decline in success rates as students progressed through the problem-solving stages, from 69% successfully constructing a correct mathematical model to only 3% accurately stating the final answer. This pattern supports Anderson et al.'s (2014) view that mathematical problem solving is sequential and that early-stage failures have cascading effects on subsequent steps. Only one of 29 students successfully identified and recorded both known and unknown information, indicating weaknesses in basic mathematical literacy, which Kilpatrick (2001) describes as essential for solving complex problems. This aligns with Febriyanti et al. (2021), who note that early-stage difficulties are often undiagnosed by teachers, leading to misaligned interventions. Consequently, weaknesses in interpretation and representation form a fragile foundation, hindering success in formula application and final evaluation (Lesh, 1981; Rushton, 2018).

The cumulative nature of these difficulties reflects not only challenges in advanced trigonometric understanding but also the shift from procedural skills to conceptual comprehension. Weber (2005) found that many students understand trigonometric functions procedurally without connecting them to their underlying concepts, a finding consistent with Nordlander's (2022) research on trigonometric limits. Cetin (2015) emphasizes that conceptual misconceptions can hinder the accurate application of formulas, even when the initial mathematical models are correct. This gap suggests that instruction focused solely on procedures, without strengthening basic mathematical literacy, is unlikely to produce mastery. Accordingly, error analysis (Rushton, 2018) offers a valuable diagnostic tool for identifying obstacles at the outset, enabling targeted teacher support. Such strategies can help minimize cumulative difficulties, enabling students to solve problems accurately and meaningfully through to completion.

NVivo *word cloud* analysis enriched the quantitative findings by highlighting keywords such as "manipulatives," "visual," "difficult," and "confused," which reflect cognitive barriers when learning relies solely on abstract explanations. This is consistent with Su and Chang (2024) and Chu et al. (2024), who demonstrate the effectiveness of *word cloud* analysis in uncovering latent thematic patterns in qualitative data. In contrast, students exhibited greater engagement and motivation when visual and interactive methods were employed, supporting Puchner et al.'s (2008) argument that physical manipulatives can enhance the representation of mathematical concepts. Similarly, Moyer-Packenham and Westenskow (2013) found that virtual manipulatives significantly enhance learning outcomes and conceptual understanding. Thus, the integration of visual media and manipulatives not only facilitates information transfer but also reduces cognitive load in complex topics such as trigonometry.

The *word cloud* analysis also indicated that students face dual challenges: conceptual difficulties, including understanding definitions and relationships among concepts, and procedural difficulties, such as selecting appropriate formulas and performing calculations. This aligns with the framework proposed by Hiebert and Lefevre (2013) and Rittle-Johnson and Alibali (1999), which stresses the interdependence of conceptual and procedural knowledge. Rittle-Johnson, Fyfe, and Loehr (2016) further assert that an imbalance between these two domains can exacerbate misconceptions. Supandi et al. (2021) similarly observe that unaddressed early-stage barriers can trigger recurring cycles of failure. In trigonometry learning, the interaction between conceptual and procedural difficulties can hinder the development of higher-order reasoning. Therefore, instructional strategies that integrate error analysis with manipulative-based interventions are essential to breaking these cycles of failure.

The implications of these findings are significant for mathematics teaching practice. First, there is an urgent need to shift pedagogical approaches from formula-centered instruction toward conceptually oriented learning supported by visual aids, reflective discussion, and contextual problem-solving exercises (Anita & Hakim, 2022). Teachers can employ error analysis as a diagnostic tool to identify specific weaknesses and design targeted interventions, as also suggested in prior studies on critical thinking in trigonometry (Ichsan et al., 2024; Jusniani, 2018). For instance, if most students struggle with applying the formula, the instructional focus should be on strengthening procedural fluency while consistently linking it to underlying conceptual understanding (Jatisunda & Nahdi, 2019; Novianti & Riajanto, 2021). Longitudinal studies would also be valuable to track the development of these skills over time in response to specific instructional strategies.

Nevertheless, this study has certain limitations. It employs an intrinsic case study design with a relatively small sample size (29 students) from a single school (SMAN 1 Kadipaten), which limits the generalizability of its results. Its focus on trigonometry also suggests that the dynamics of critical thinking difficulties may differ across other mathematical topics. Future research should replicate this study with larger and more diverse samples to test the generalizability of its findings. Additionally, experimental research could be conducted to evaluate the effectiveness of visual and interactive teaching interventions in improving students' critical thinking skills in measurable ways. Longitudinal studies would also be valuable to track the development of these skills over time in response to specific instructional strategies.

4. CONCLUSION

This study aimed to explore the characteristics of students' critical thinking skills in solving trigonometric problems through a qualitative approach based on answer analysis and interviews. The findings revealed that the majority of students encountered difficulties in meeting the critical thinking indicators, particularly in identifying essential information, accurately applying formulas, and evaluating final answers. Out of a total of 29 students, only 14 were able to thoroughly read and comprehend the problem statements. Twenty students successfully constructed appropriate mathematical models, yet only 9 were able to perform correct calculations. Only one student demonstrated the ability to understand and write the final answer completely and accurately. These results highlight that, although some students exhibited potential for analytical thinking, there remain serious weaknesses in procedural and evaluative skills, which are integral components of critical thinking. The primary barriers were identified as a lack of understanding of fundamental trigonometric concepts, limited use of effective problem-solving strategies, and weak self-regulation in reviewing the problem-solving process. Consequently, this study concludes that strengthening critical thinking skills in trigonometry learning cannot rely solely on repetitive practice exercises but requires strategic and contextual pedagogical interventions. The practical implications suggest the importance of employing visual aids, problem-based learning approaches, and higher-order questioning techniques to stimulate idea exploration, reflection, and more mature decision-making. Future research is recommended to develop diagnostic assessment tools based on critical thinking indicators and to evaluate the effectiveness of various innovative learning models in enhancing students' critical thinking skills in abstract and complex mathematical topics such as trigonometry.

5. REFERENCES

Afriansyah, E. A., Herman, T., & Dahlan, J. A. (2021, February). Critical thinking skills in mathematics. In *Journal of Physics: Conference Series* (Vol. 1778, No. 1, p. 012013). IOP Publishing. https://doi.org/10.1088/1742-6596/1778/1/012013

Ahmad, H., Al Yakin, A., & Sarbi, S. (2018, November). The analysis of student error in solve the problem of spherical trigonometry application. In *Journal of Physics: Conference Series* (Vol. 1114, No. 1, p. 012114). IOP Publishing. https://doi.org/10.1088/1742-6596/1114/1/012114

Anderson, J. R., Lee, H. S., & Fincham, J. M. (2014). Discovering the structure of mathematical problem solving. *NeuroImage, 97,* 163–177. https://doi.org/10.1016/j.neuroimage.2014.04.031

Angraini, N., & Masykur, R. (2018). Modul matematika berdasarkan model pembelajaran problem based learning materi pokok trigonometri. *Desimal: Jurnal Matematika, 1*(2), 217–228. https://doi.org/10.24042/djm.v1i2.2558

Anita, A., & Hakim, D. L. (2022). Analisis kesulitan siswa SMP dalam menyelesaikan soal kemampuan berpikir kritis matematis. *PHI: Jurnal Pendidikan Matematika, 6*(2), 175. https://doi.org/10.33087/phi.v6i2.242

Braun, H. I., Shavelson, R. J., Zlatkin-Troitschanskaia, O., & Borowiec, K. (2020, September). Performance assessment of critical thinking: Conceptualization, design, and implementation. In *Frontiers in Education* (Vol. 5, p. 156). Frontiers Media SA. https://doi.org/10.3389/feduc.2020.00156

Cáceres, M., Nussbaum, M., & Ortiz, J. (2020). Integrating critical thinking into the classroom: A teacher's perspective. *Thinking Skills and Creativity, 37,* 100674. https://doi.org/10.1016/j.tsc.2020.100674

Cahya, E., & Juandi, D. (2021). Students' critical thinking skills in solving mathematical problems: A systematic procedure of grounded theory study. *International Journal of Instruction, 14*(4), 529–548. https://files.eric.ed.gov/fulltext/EJ1319103.pdf

Cetin, O. F. (2015). Students' perceptions and development of conceptual understanding regarding trigonometry and trigonometric function. *Educational Research and Reviews, 10*(3), 338–350. https://doi.org/10.5897/ERR2014.2017

Chakravarty, U., Hariharanadh, S., & Hemavathi, D. (2023). Student feedback review analyzer using sentiment analysis. In *Proceedings* (pp. 525–528). https://doi.org/10.13052/rp-9788770040723.104

Chu, H. C., Hwang, G. J., & Chang, C. Y. (2024). An integrative review with word cloud analysis of computer-supported collaborative learning. *Journal of Science Education and Technology,* 1–14. https://doi.org/10.1007/s10956-024-10156-2

Creswell, J. W., & Poth, C. N. (2018). *Qualitative inquiry and research design: Choosing among five approaches* (4th ed.). SAGE Publications.

Dwyer, C. P. (2023). An evaluative review of barriers to critical thinking in educational and real-world settings. *Journal of Intelligence, 11*(6), 105. https://doi.org/10.3390/jintelligence11060105

Facione, P. A. (2011). *Critical thinking: What it is and why it counts*. Insight Assessment.

Fauzi, F., Tuhuteru, L., Sampe, F., Ausat, A. M. A., & Hatta, H. R. (2023). Analysing the role of ChatGPT in improving student productivity in higher education. *Journal on Education, 5*(4), 14886–14891. https://doi.org/10.31004/joe.v5i4.2563

Febriyanti, R., Mustadi, A., & Jerussalem, M. A. (2021). Students' learning difficulties in mathematics: How do teachers diagnose and how do teachers solve them. *Jurnal Pendidikan Matematika, 15*(1), 23–36. https://doi.org/10.22342/jpm.15.1.10564.23-36

Firdaus, F., Kailani, I., Bakar, M. N. B., & Bakry, B. (2015). Developing critical thinking skills of students in mathematics learning. *Journal of Education and Learning (EduLearn), 9*(3), 226–236. https://doi.org/10.11591/edulearn.v9i3.1830

Fitriani, F., & Kowiyah. (2022). Mathematics critical thinking skills for the third grade elementary school students on fractions material. *MIMBAR PGSD Undiksha, 10*(3), 463–468. https://doi.org/10.23887/jjpgsd.v10i3.48741

Franklin, E. I., Iwu, C. G., & Dubihlela, J. (2022). Students' views regarding the barriers to learning critical thinking. *International Journal of Research in Business and Social Science, 11*(4), 355–364. https://doi.org/10.20525/ijrbs.v11i4.1797

Hamzah, N., Maat, S. M., & Ikhsan, Z. (2021). A systematic review on pupils' misconceptions and errors in trigonometry. *Pegem Journal of Education and Instruction, 11*(4), 209–218. https://doi.org/10.47750/pegegog.11.04.20

Hidayati, U. (2020). Analysis of student errors in solving trigonometry problems. *JME (Journal of Mathematics Education), 5*(1), 54–60. https://core.ac.uk/download/pdf/328111412.pdf

Hiebert, J., & Lefevre, P. (2013). Conceptual and procedural knowledge in mathematics: An introductory analysis. In *Conceptual and procedural knowledge* (pp. 1–27). Routledge. https://doi.org/10.4324/9780203063538-1

Ichsan, Mulyati, Duma, S. Y., & Nirfayanti. (2024). The effectiveness of mathematics learning with the STEAM approach in improving students' critical thinking skills. *Journal Name,* 1(September), 1–16.

Jablonka, E. (2014). Critical thinking in mathematics education. In *Encyclopedia of mathematics education* (pp. 121–125). Springer. https://doi.org/10.1007/978-94-007-4978-8_35

Jatisunda, M. G., & Nahdi, D. S. (2019). Kesulitan siswa dalam memahami konsep trigonometri dilihat dari learning obstacles. *Jurnal Didactical Mathematics,* 2(1), 9–16. http://download.garuda.kemdikbud.go.id/article.php?article=1254409&val=15073&title=Kesulitan%20Siswa%20Dalam%20Memahami%20Konsep%20Trigonometri%20Di%20Lihat%20Dari%20Learning%20Obstacles

Jusniani, N. (2018). Analisis kesalahan jawaban siswa pada kemampuan pemahaman matematis melalui pembelajaran kontekstual. *Prisma, 7*(1), 82–90. https://jurnal.unsur.ac.id/prisma/article/download/361/274

Kilpatrick, J. (2001). Understanding mathematical literacy: The contribution of research. *Educational Studies in Mathematics, 47*(1), 101–116. https://doi.org/10.1023/A:1017973827514

Kükey, E., & Aslaner, R. (2023). To what extent can teachers and preservice teachers predict students' mathematical thinking? A qualitative analysis. *Journal of Pedagogical Research, 7*(4), 91–110. https://doi.org/10.33902/JPR.202319993

Kvale, S., & Brinkmann, S. (2009). *InterViews: Learning the craft of qualitative research interviewing* (2nd ed.). SAGE Publications.

Lesh, R. (1981). Applied mathematical problem solving. *Educational Studies in Mathematics, 12*(2), 235–264. https://doi.org/10.1007/BF00305624

Lopezosa, C. (2020). Entrevistas semiestructuradas con NVivo: Pasos para un análisis cualitativo eficaz. *Methodos: Anuario de Métodos de Investigación en Comunicación Social, 1*, 88–97. https://doi.org/10.31009/methodos.2020.i01.08

Malambo, P. (2021). Implicit misconceptions in prospective mathematics teachers' reasoning about trigonometric concepts. *Contemporary Mathematics and Science Education, 2*(2), ep21011. https://doi.org/10.30935/conmaths/11054

Maulidiya, M., & Nurlaelah, E. (2019, February). The effect of problem-based learning on critical thinking ability in mathematics education. In *Journal of Physics: Conference Series* (Vol. 1157, No. 4, p. 042063). IOP Publishing. https://doi.org/10.1088/1742-6596/1157/4/042063

Merriam, S. B., & Tisdell, E. J. (2016). *Qualitative research: A guide to design and implementation* (4th ed.). Jossey-Bass.

Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). *Qualitative data analysis: A methods sourcebook* (3rd ed.). SAGE Publications.

Moyer-Packenham, P. S., & Westenskow, A. (2013). Effects of virtual manipulatives on student achievement and mathematics learning. *International Journal of Virtual and Personal Learning Environments, 4*(3), 35–50. https://doi.org/10.4018/jvple.2013070103

Müller de Andrade, A., do Amaral, M. R., & Monteiro, C. (2020). NVivo and the building of discourse analysis categories in qualitative research. *Revista Brasileira de Educação, 25*, 1–22. https://doi.org/10.1590/s1413-24782020250090

Müller de Andrade, D., Brandão Schmidt, E., & Celente Montiel, F. (2020). Uso do software NVivo como ferramenta auxiliar da organização de informações na análise textual discursiva. *Revista Pesquisa Qualitativa, 8*(19), 948–970. https://doi.org/10.33361/rpq.2020.v.8.n.19.357

Nanmumpuni, H. P., & Retnawati, H. (2021, February). Analysis of senior high school students' difficulty in resolving trigonometry conceptual problems. In *Journal of Physics: Conference Series* (Vol. 1776, No. 1, p. 012012). IOP Publishing. https://doi.org/10.1088/1742-6596/1776/1/012012

Nordlander, M. C. (2022). Lifting the understanding of trigonometric limits from procedural towards conceptual. *International Journal of Mathematical Education in Science and Technology, 53*(11), 2973–2986. https://doi.org/10.1080/0020739X.2021.1927226

Novianti, V., & Riajanto, M. L. E. J. (2021). Analisis kesulitan siswa SMK dalam menyelesaikan soal materi trigonometri. *JPMI (Jurnal Pembelajaran Matematika Inovatif), 4*(1), 161–168. https://doi.org/10.22460/jpmi.v4i1.p%25p

Nurmeidina, R., & Rafidiyah, D. (2019, August). Analysis of students' difficulties in solving trigonometry problems. In *PROFUNEDU 2019: Proceedings of the 4th Progressive and Fun Education International Conference, 6–8 August 2019, Makassar, Indonesia* (p. 9). https://www.academia.edu/download/78252379/eai.7-8-2019.pdf

Obeng, B. A., Banson, G. M., Owusu, E., & Owusu, R. (2024). Analysis of senior high school students' errors in solving trigonometry. *Cogent Education, 11*(1), 2385119. https://doi.org/10.1080/2331186X.2024.2385119

Ongesa, C. M. (2020). The critical thinking skill gap in the Kenyan education curriculum: The 21st century skills for the global citizen. *Journal of Interdisciplinary Studies in Education, 9*(SI), 178–191. https://doi.org/10.32674/jise.v9iSI.1860

Özkaya, Ö. M., & Semerci, N. (2022). The effects of learning activities on eliminating learners' in-class barriers to critical thinking. *Bartın University Journal of Faculty of Education, 11*(3), 737–757. https://doi.org/10.14686/buefad.1120124

Pakpahan, G. M. B., Aziz, T. A., & Ambarwati, L. (2023). Identification of critical thinking skills in mathematics students of class VIII SMPN 61 West Jakarta. *Math Didactic: Jurnal Pendidikan Matematika, 9*(1), 98–109. https://doi.org/10.33654/math.v9i1.2102

Pangestika, W. A., & Faiziyah, N. (2022). Students' mathematics power viewed from the students' critical thinking skills. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika, 11*(3), 1897. https://doi.org/10.24127/ajpm.v11i3.5322

Puchner, L., Taylor, A., O'Donnell, B., & Fick, K. (2008). Teacher learning and mathematics manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School Science and Mathematics, 108*(7), 313–325. https://doi.org/10.1111/j.1949-8594.2008.tb17844.x

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology, 91*(1), 175–189. https://doi.org/10.1037/0022-0663.91.1.175

Rittle-Johnson, B., Fyfe, E. R., & Loehr, A. M. (2016). Improving conceptual and procedural knowledge: The impact of instructional content within a mathematics lesson. *British Journal of Educational Psychology, 86*(4), 576–591. https://doi.org/10.1111/bjep.12124

Rushton, S. J. (2018). Teaching and learning mathematics through error analysis. *Fields Mathematics Education Journal, 3*(1), 1–12. https://doi.org/10.1186/s40928-018-0009-y

Sachdeva, S., & Eggen, P. O. (2021). Learners' critical thinking about learning mathematics. *International Electronic Journal of Mathematics Education, 16*(3), em0644. https://doi.org/10.29333/iejme/11003

Shavelson, R. J., Zlatkin-Troitschanskaia, O., Beck, K., Schmidt, S., & Marino, J. P. (2019). Assessment of university students' critical thinking: Next generation performance assessment. *International Journal of Testing, 19*(4), 337–362. https://doi.org/10.1080/15305058.2018.1543309

Siahaan, Y. L. O., & Meilani, R. I. (2019). Sistem kompensasi dan kepuasan kerja guru tidak tetap di sebuah SMK swasta di Indonesia. *Jurnal Pendidikan Manajemen Perkantoran, 4*(2), 141–150. https://doi.org/10.17509/jpm.v4i2.18008

Su, W. S., & Chang, C. Y. (2024). An integrative review with word cloud analysis of STEM education. *Journal of Science Education and Technology*, 1–14. https://doi.org/10.1007/s10956-024-10134-8

Supandi, S., Suyitno, H., Sukestiyarno, Y. L., & Dwijanto, D. (2021, June). Learning barriers and student creativity in solving math problems. In *Journal of Physics: Conference Series* (Vol. 1918, No. 4, p. 042088). IOP Publishing. https://doi.org/10.1088/1742-6596/1918/4/042088

Syafril, S., Aini, N. R., Netriwati, Pahrudin, A., Yaumas, N. E., & Engkizar. (2020). Spirit of mathematics critical thinking skills (CTS). *Journal of Physics: Conference Series, 1467*(1), 012069. https://doi.org/10.1088/1742-6596/1467/1/012069

Tran, D., & O'Connor, B. R. (2024). Teacher curriculum competence: How teachers act in curriculum making. *Journal of Curriculum Studies, 56*(1), 1–16. https://doi.org/10.1080/00220272.2023.2271541

Weber, K. (2005). Students' understanding of trigonometric functions. *Mathematics Education Research Journal, 17*(3), 91–112. https://doi.org/10.1007/BF03217423

Woods, M., Paulus, T., Atkins, D. P., & Macklin, R. (2016). Advancing qualitative research using qualitative data analysis software (QDAS)? Reviewing potential versus practice in published studies using ATLAS.ti and NVivo, 1994–2013. *Social Science Computer Review, 34*(5), 597–617. https://doi.org/10.1177/0894439315596311

Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). SAGE Publications.