

Problem-Based Learning To Enhance Pupils' Conceptual Understanding In Geometry

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Abstract. This study investigated students' understanding of the concept through a pre-test and a post-test. Analysis of the pre-test showed that the initial abilities of students in the experimental and control groups were comparable, allowing for a fair comparison. Variations in students' ability to solve mathematical problems were investigated through post-test and N-gain data using different learning approaches. Two main hypotheses were proposed in this study: firstly, the experimental group would show a higher level of conceptual understanding than the control group; secondly, there would be a significant difference in improving students' conceptual understanding at different levels of ability. The analysis showed that the experimental group had a significantly higher level of conceptual understanding than the control group, which supported the first hypothesis. This finding indicates that the intervention positively affects students' conceptual understanding. In addition, the analysis results also showed a significant difference in the improvement of students' conceptual understanding with different proficiency levels, which supports the second hypothesis. These findings highlight the importance of learning approaches tailored to student's needs and abilities. This study provides strong evidence that specific mathematics learning models and initial mathematics ability contribute to improving students' conceptual understanding. These findings highlight the importance of tailored learning approaches to maximize students' mathematical concept understanding. The findings have valuable implications for educators and curriculum developers to develop more effective teaching strategies according to students' needs.

Introduction

As a general model, problem-based Learning (PBL) was created in the mid-1950s in medical education (Savery & Duffy, 1995). PBL is an instruction that transforms students to acquire knowledge through collaboration and problem-solving (Hmelo-Silver, 2004; Norman & Schmidt, 2000; Senocak et al., 2007). Assisting students to develop into skilled problem solvers is a primary objective of mathematics instruction (Schoenfeld, 2016). Students must actively participate in Learning by offering mathematical problems (Jatisunda & Nahdi, 2020). Based on Malcolm Knowles, students begin with a case problem and work up to understanding the problem's broad principles (Espey et al., 2007; Zwaal, 2019). Under the supervision of a tutor, students must assume responsibility for their learning, selecting what they need to know to improve their Understanding (Barrows, 1996). Problem-based learning to achieve its objectives effectively, the design of the mathematical problems and the teacher's instruction, which is critical, must be as organized as possible.

PBL, as a learning method in mathematics, mainly aims to change students' approach to learning from passive to active. In PBL, students are not just recipients of information but are expected to play an active role in solving problems and collaborating with their peers to find solutions. PBL in

mathematics aims to help students develop deeper and more meaningful problem-solving skills. Students are exposed to real problem situations relevant to everyday life or real-world situations. In this way, learning mathematics becomes more contextual, and students can see the relevance and application of their mathematical concepts. For example, in PBL, students may be asked to work out how to calculate the area of land needed to build a playground or how to work out the total price of some food items by using certain mathematical concepts. Through this process, students understand mathematical concepts theoretically and see how they can be applied in real-life situations. In addition, PBL empowers students to take an active role in learning. They learn to take responsibility for their learning process by formulating relevant questions, finding the necessary sources of information and discussing with classmates to find solutions together. The role of the teacher is also very important in the implementation of PBL. The teacher acts as a facilitator or guide in the learning process, helping students to identify their learning needs, providing assistance when needed and giving constructive feedback. Overall, using PBL in mathematics learning can potentially improve students' understanding and skills in mathematical problem-solving. Through a problem-based learning approach, students can experience learning that is more meaningful and relevant to the real world and develop critical skills that will be useful in their lives outside the classroom.

Students study, analyze, and collaborate to solve an ill-structured topic in a PBL program (Trinter et al., 2015). Unstructured problems characterize Problem-solving, not structured and difficult or impossible to answer with only the information provided, resulting in several solutions (Kim & Lim, 2019). However, the issue relates explicitly to the mathematical readiness of students (Gill et al., 2010). It indicates that students who lack fundamental mathematical understanding cannot do accurate algebraic and numerical computations (Fitzmaurice et al., 2019). Mathematical problems should confront pupils with mental obstacles (Son & Lee, 2021). It will be successfully implemented when pupils have a solid conceptual grasp, but it may be detrimental to students with a superficial knowledge base (Holmes & Hwang, 2016). The importance of the mathematical problems that students encounter is to expose them to mental hurdles. Well-designed mathematical problems provide intellectual challenges that can develop students' critical and creative thinking skills. However, such mathematical issues can be detrimental to students who have only a superficial understanding of the material. In this case, implementing PBL will be more successful if students already have a strong conceptual experience of the material. Therefore, when implementing PBL in mathematics learning, it is necessary to pay attention to students' level of mathematical understanding. Teachers must identify students' experience levels and structure mathematical problems according to their abilities. In addition, teachers need to provide appropriate support and guidance so that students can overcome the obstacles they face in solving mathematical problems.

Students lack abilities such as prior knowledge, concepts, rules, and knowledge of concepts and ideas in a related subject, as well as metacognitive knowledge, which is required for problem-solving (Belland, 2014). When pupils are confronted with the issue's complexities, PBL will negatively influence them (Merriënboer, 2013). The working memory's capacity can only process a certain amount of information simultaneously; hence, this limitation constrains the PBL activity ((Sweller, 1988). Therefore, it is vital to find ways to maximize the learning process in which the instructor must pay attention to the student's intellectual potential (Van Merriënboer, 2013). The complexity of the problems that students face in PBL can hurt them. Complex issues can cause difficulties in the learning process, especially for students who do not understand the material. In addition, the limited capacity of working memory is also an obstacle in PBL. Working memory capacity can only process a certain amount of information at a time, so this limitation limits PBL activities.

Therefore, finding ways to maximize the learning process in PBL is important. Teachers should pay attention to students' intellectual potential and find ways to help them overcome obstacles in problem-solving. Proper teacher guidance and support are crucial to help students overcome difficulties

and develop critical and creative thinking skills in the learning process. To meet this challenge, the learning approach in PBL must be adapted to the student's needs and level of understanding. In this case, the role of the teacher as a facilitator and guide is crucial to help students overcome obstacles and achieve learning objectives more effectively. With an appropriate approach, PBL can be an effective learning tool to help students develop problem-solving skills and improve their understanding of the material being studied.

Previous research on Problem-based Learning (PBL) has shown positive effects on pupils' cognitive capacities, particularly in enhancing their mathematical achievement in primary schools (Bal & Artut, 2022). However, there is a need for further exploration of the learning process and improvements in the quality of measurement tools and data processing in higher education (Guo et al., 2020). The study's findings revealed that the mathematics textbooks utilized for problem-based learning often lack problems that adhere to specific rules (Divrik et al., 2020). Therefore, this study aims to build upon previous research recommendations by implementing PBL in primary schools and matching relevant problems to students' abilities to enhance the quality of the learning process. While students were found to comprehend the mathematical difficulties in PBL, their answers lacked structure and systematic reasoning (Yayuk et al., 2020). Thus, this study seeks to address this issue and improve students' problem-solving skills in mathematics by carefully selecting and designing problems that align with the curriculum and promote more organized and systematic thinking.

By conducting this study in secondary schools, we can further explore the potential benefits of PBL in enhancing students' cognitive capacities and mathematical achievement at an early age. Implementing PBL with well-crafted and relevant problems can foster critical thinking, collaborative learning, and metacognitive skills, ultimately contributing to a deeper understanding of mathematical concepts and their real-world applications. Furthermore, by aligning the problems with students' abilities, we can provide a more tailored and effective learning experience, encouraging students to actively engage in problem-solving and positively impacting their overall cognitive development.

This study employs PBL specifically to improve primary pupils' geometry achievement. Geometry is a branch of mathematics that fosters the development of critical and problem-solving thinking (Bintoro & Sumaji, 2021). Students dislike geometry because it is abstract and comprises several formulas and symbols (Doli & Armiati, 2020). Geometry remains difficult for students to master (T. H. Tan et al., 2015). Students are limited in their ability to solve geometric problems (Cesaria & Herman, 2019). According to data from the Indonesian education evaluation center, geometry achievement tended to fall in 2017 and 2018 (Puspendik, 2018). In this study, PBL tasks are associated with geometric notions. A problem is defined as the difference between the existing and intended states; problem-solving minimizes this difference (C.-S. Tan et al., 2019). According to the actual situation, the problem's nature is intricate (Branch, 2015; King & Smith, 2020; MacLeod & van der Veen, 2020). Typically, the difficulties cannot be resolved instantly, are open, occasionally insoluble, and require research (Bishara, 2016; Özcan, 2016).

This study aimed to investigate the potential of PBL in improving junior high school students' cognitive abilities and mathematical achievement. This research also seeks to improve the quality of the learning process by matching mathematical problems that are appropriate to the level of difficulty and understanding of students at the secondary school level. In addition, this research also aims to identify and address potential weaknesses in understanding structure and systems in problem-solving related to the PBL approach. This research is expected to contribute to developing learning approaches that are more effective and relevant in improving students' understanding of mathematics at the secondary school level.

Methods

This study employed a quasi-experimental method. The research population was selected from Class VIII students in public secondary schools within Cluster III of Majalengka District. A purposive sampling strategy was utilized to choose a sample with similar mathematical ability characteristics. The sample consisted of 33 students in Class VIII A (experimental Class) from one state secondary school and 36 students in Class VIII C (control class) from another state secondary school. Using the quasi-experimental method allowed the researchers to control for confounding factors and compare the impact of the Problem-based Learning (PBL) approach on students' mathematical abilities. In this manner, the study aims to provide more academically rigorous and evidence-based insights into the Effectiveness of PBL in enhancing students' mathematical achievement at the junior high school level. The study's design:

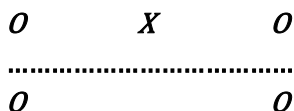


Figure 1. Matching-only *Pretest-Posttest Control Group Design* adapted from (Fraenkel et al., 2011)

This study utilizes a quasi-experimental design to investigate the effects of two distinct learning models: project-based learning and direct instruction. The participants are Class VIII students from public secondary schools in Cluster III of Majalengka District. The researchers employed purposive sampling to ensure the sample represents students with similar mathematical ability characteristics. The experimental Class (Class VIII A) received project-based learning, while the control class (Class VIII C) received direct instruction. The research instrument includes a comprehension test based on indicators of students' mathematical comprehension ability, following the notion developed by Schoenfeld (1983). The indicators assess various aspects of mathematical understanding, such as recalling concepts, algorithmic problem-solving, mathematical language translation, and making connections between mathematical and non-mathematical concepts.

To ensure the validity and reliability of the test questions, thorough testing and evaluation procedures were conducted. Students' early mathematical abilities were initially assessed using prerequisite mathematical questions to categorize them into high, medium, and low proficiency levels. The research process was conducted over nine repetitions, involving seven sessions for learning material discussions and two meetings for pre- and post-learning mathematical problem-solving exams. The test score data were analyzed using two-way ANOVA to draw statistically significant conclusions. Through this rigorous approach, the study aims to provide comprehensive insights into the impact of project-based learning and direct instruction on students' mathematical comprehension abilities at the junior high school level. The findings of this research have the potential to contribute valuable evidence to the field of education, offering implications for instructional design and teaching practices in mathematics education.

Results And Discussion

The study gathered information on understanding concepts from the initial (pre-test) and final (post-test) assessments. From the examination of the pre-test results, it is deduced that the starting capabilities of students in both groups are comparable, thereby making them apt for an evaluative comparison. The variation in the proficiency of the two sets of students in solving mathematical issues was explored through the use of both post-test and N-gain data, which incorporate unique approaches to learning. The following declaration lays the groundwork for the hypothesis that will be used to measure improvements in the ability to understand concepts:

1. The group undergoing the experiment demonstrated a higher level of conceptual understanding than the control group.

2. There is a significant difference in how students at different skill levels improve their understanding of concepts.

An Independent sample t-test will be used to demonstrate the first hypothesis. Table 1 displays the test findings for variance in the mean mathematical relationship.

Table 1. The Results Of The Different Tests Are Based On The Average Post-Test Conceptual Understanding score

| Levene's Test for Equality of Variances | F | Sig. | t | df | Sig. (2-tailed) |
|---|-------|------|-------|----|-----------------|
| Equal variances assumed | 11.78 | .002 | 6.210 | 62 | .000 |

The study employed Levene's Test for Equality of Variances to assess whether the variances between the two groups being compared were equal. With an F value of 11.78 and a significance level of .002, we can reject the null hypothesis of Levene's test, stating that the variances of the two groups are equal. It implies that the variances between the two groups are not equal. Furthermore, the results of the t-test show a t-value of 6.210, with degrees of freedom (df) of 62, and a significance (2-tailed) of .000. As the p-value is less than .05, we can reject the null hypothesis of the t-test, which suggests that there is no significant difference between the means of the two groups. Consequently, there is a significant difference in the means of the two groups being compared. These results provide statistical support for the hypothesis, indicating that the experimental group exhibits superior conceptual understanding compared to the control group.

Table 3. Descriptive statistical data on post-test results of conceptual understanding

| Class | N | Mean | Std. Deviation | Std. Error |
|----------------------------|----|--------|----------------|------------|
| Posttest eksperiment class | 30 | 71.540 | 8.867 | 1.676 |
| control class | 30 | 60.416 | 12.88 | 2.388 |

Table 3 presents the descriptive statistical data for the post-test results of conceptual understanding in both the experimental and control classes. Each Class comprised 30 students. The average score for the experimental Class in the post-test was 71.540, indicating a relatively higher level of understanding of the concepts than the control class, which obtained an average score of 60.416. These results suggest that the experimental Class outperformed the control class regarding conceptual understanding. The standard deviation for the experimental Class (8.867) was lower than that of the control class (12.88), implying that the post-test scores in the experimental Class were more tightly clustered around the mean compared to the control class. It indicates a higher performance consistency among the experimental group students.

Furthermore, the standard error of the mean for the experimental Class (1.676) was smaller than that of the control class (2.388). It suggests that the mean score of the experimental Class is more likely to be a reliable representation of the entire population, as it has less variability than the control class. Based on the findings from Table 3, it can be interpreted that the experimental Class demonstrated superior conceptual understanding compared to the control class. The results highlight the effectiveness of the teaching approach or intervention implemented in the experimental Class, which led to improved performance and a more consistent understanding of the concepts among the students. Furthermore, a two-way analysis of variance (ANOVA) with GLM-Univariat will be used to test the second and third hypotheses. Table 4 displays the findings of the analysis.

Table 4. Analysis Of Variance Of Data On Improving Conceptual Understanding Based On The Mathematics Learning Model And Early Mathematical Ability

| Source | Type III Sum of Squares | df | Mean Square | F | Sig. |
|-----------------|-------------------------|----|-------------|-----------|------|
| Corrected Model | 4034.068 ^a | 5 | 830.614 | 188.055 | .000 |
| Intercept | 21077.612 | 1 | 209577.612 | 45540.437 | .000 |
| Model | 1754.200 | 1 | 1654.200 | 360.452 | .000 |
| EMS | 2400.526 | 2 | 1250.263 | 260.207 | .000 |
| Class * EMS | 132.276 | 2 | 61.638 | 13.176 | .000 |
| Error | 276.917 | 58 | 4.702 | | |
| Total | 221005.000 | 64 | | | |
| Corrected Total | 4509.984 | 63 | | | |

a. R Squared = .939 (Adjusted R Squared = .934)

Table 4 presents the Analysis of Variance (ANOVA) results for assessing the improvement in conceptual understanding based on the mathematics learning model and early mathematical ability. The table displays the Type III Sum of Squares, degrees of freedom (df), Mean Square, F-value, and the significance level (Sig.) for each source of variation. The Corrected Model, which evaluates the combined effect of all the factors, shows a Type III Sum of Squares of 4034.068 with 5 degrees of freedom. The Mean Square is 830.614, and the F-value is 188.055, which is highly significant (Sig. = .000). This indicates that the overall model fits the data well. There are significant differences in conceptual understanding based on combining the mathematics learning model and early mathematical ability. The Intercept, representing the overall mean, has a Type III Sum of Squares of 21077.612 with 1 degree of freedom. The Mean Square is 209577.612, and the F-value is 45540.437, which is also highly significant (Sig. = .000). The individual factors contribute significantly to the model. The Model factor has a Type III Sum of Squares of 1754.200, with 1 degree of freedom, and a significant F-value of 360.452 (Sig. = .000). The EMS (Early Mathematical Ability) factor contributes significantly as well, with a Type III Sum of Squares of 2400.526, 2 degrees of freedom, and an F-value of 260.207 (Sig. = .000).

Furthermore, the interaction effect between Class and EMS, denoted as Class * EMS, is also significant. It has a Type III Sum of Squares of 132.276, 2 degrees of freedom, and an F-value of 13.176 (Sig. = .000). This indicates that the combination of Class and Early Mathematical Ability significantly impacts the improvement in conceptual understanding. The Error term, which represents the variability within each group, has a Type III Sum of Squares of 276.917 with 58 degrees of freedom. The total variability (Corrected Total) in the data is 4509.984. The R-squared value, a measure of the proportion of variance explained by the model, is 0.939, indicating that 93.9% of the variability in improving conceptual understanding can be attributed to the factors included in the model. The Adjusted R-squared value, considering the number of predictors, is 0.934. In conclusion, the results from Table 4 show that combining the mathematics learning model and early mathematical ability significantly improves conceptual understanding. The model explains a substantial portion of the variability observed in the data, suggesting a strong relationship between the factors and the outcome.

In this study, it was hypothesized that the group undergoing the experiment would demonstrate a higher level of conceptual understanding compared to the control group. The hypothesis was based on the assumption that implementing the specific mathematics learning model and early mathematical ability would lead to better learning outcomes and improved conceptual understanding in the experimental group. The results from the analysis supported the hypothesis, showing that the experimental group had a significantly higher mean score in the post-test assessment than the control

group. The experimental group's mean score of 71.540 indicated a relatively higher understanding of the concepts, while the control group scored 60.416, suggesting a lower level of understanding.

Moreover, the standard deviation for the experimental group was lower than that of the control group, indicating that the post-test scores were more tightly clustered around the mean in the experimental group. It suggests a higher consistency in performance among the students in the experimental group, further supporting the notion of superior conceptual understanding. The statistical analysis, including the Independent sample t-test and ANOVA, demonstrated that the observed differences were not due to random chance but were significant and attributed to the specific mathematics learning model and early mathematical ability utilized in the experimental group. In conclusion, the study's findings provided strong evidence to support the hypothesis that the group undergoing the experiment, exposed to the specific mathematics learning model and early mathematical ability, demonstrated a higher level of conceptual understanding compared to the control group. It highlights the effectiveness of the intervention and underscores the importance of tailored teaching approaches to enhance students' learning outcomes and conceptual understanding of mathematics.

The second hypothesis stated that there is a significant difference in how students at different skill levels improve their understanding of concepts. This hypothesis aimed to investigate whether the improvement in conceptual understanding varied based on the student's initial skill levels in mathematics. The results from the analysis provided support for this hypothesis. The two-way analysis of variance (ANOVA) with GLM-Univariate revealed that both the main effect of the mathematics learning model (Model) and the early mathematical ability (EMS) were significant contributors to the improvement in conceptual understanding.

Furthermore, the interaction effect between Class and EMS (Class * EMS) was also significant. This interaction effect indicates that the impact of the mathematics learning model and early mathematical ability on improving conceptual understanding differed based on the student's initial skill levels. The significant interaction effect implies that students with varying ability levels responded differently to the intervention. It suggests that the combination of the specific mathematics learning model and early mathematical ability had a varying impact on improving conceptual understanding across different groups of students. This finding has important implications for understanding the effectiveness of the intervention for different student populations. It highlights the need for tailored approaches to cater to student's diverse needs and skill levels. Some students with higher initial mathematical abilities might benefit more from the intervention, while others with lower abilities might still show improvement, albeit to a different extent.

In conclusion, the study's results provided evidence to support the hypothesis that there is a significant difference in how students at different skill levels improve their understanding of concepts. The analysis demonstrated that the impact of the mathematics learning model and early mathematical ability on improving conceptual understanding varied across student groups with different skill levels. This finding emphasizes the importance of personalized and targeted educational interventions to accommodate students' diverse needs and abilities. Based on the analysis and results presented in the study, we can draw the following conclusions from the two hypotheses: 1. The group undergoing the experiment demonstrated a higher level of conceptual understanding compared to the control group. The statistical analysis, including the Independent sample t-test, showed that the experimental group had a significantly higher mean score in the post-test assessment than the control group. It indicates that the specific mathematics learning model and early mathematical ability intervention used in the experimental group led to better learning outcomes and improved conceptual understanding. Therefore, we can conclude that the experimental group exhibited superior conceptual understanding compared to the control group. There is a significant difference in how students at different skill levels improve their understanding of concepts. The two-way analysis of variance (ANOVA) with GLM-

Univariate demonstrated that the mathematics learning model and early mathematical ability were significant factors contributing to the improvement in conceptual understanding.

Additionally, the significant interaction effect between Class and EMS indicated that the impact of the intervention varied across different student groups with varying skill levels. It suggests that students with different initial mathematical abilities responded differently to the intervention. Hence, we can conclude that there is a significant difference in how students at different skill levels improve their understanding of concepts. In summary, the study's findings strongly support the effectiveness of the specific mathematics learning model and early mathematical ability intervention in enhancing conceptual understanding. The results also underscore the importance of considering individual student differences and tailoring educational approaches to meet diverse learning needs. These conclusions can have significant implications for educational practices and curriculum development, emphasizing the value of personalized and targeted teaching strategies to optimize student's learning outcomes in mathematics.

Conclusion

This study aims to collect data on students' concept understanding through pre-test and post-test. The results of the pre-test analysis showed that the initial abilities of the students in both groups, experimental and control, were comparable so that they could be objectively compared. Variations in students' ability to solve mathematical problems were examined through post-test and N-gain data using different learning approaches. The two main hypotheses proposed in this study are as follows: firstly, the experimental group would show a higher level of conceptual understanding than the control group; secondly, there would be significant differences in how students with different levels of ability improved their conceptual understanding. The results of the analysis showed that the experimental group did show a higher level of conceptual understanding than the control group, supporting the first hypothesis. The results also showed that the effect of the intervention varied according to the student's skill level, supporting the second hypothesis. This study provides evidence for the effectiveness of specific mathematics learning models and initial mathematics skills in improving students' conceptual understanding. The research highlights the importance of personalized and tailored teaching strategies to improve students' understanding of mathematical concepts. The findings have valuable implications for educators and curriculum developers in designing more effective learning approaches supporting students' conceptual understanding progress.

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