Development of Mathematics Teaching Materials Using Didactical Design Research: A Study on Enhancing Pedagogical Content Knowledge in Quadratic Inequalities

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Abstract
This study presents the development of mathematics instructional materials using the Didactical Design Research (DDR) framework, aiming to enhance the Pedagogical Content Knowledge (PCK) of prospective mathematics teachers in fractional number operations. The research adopted the ADDIE model and encompassed Analysis, Design, Development, Implementation, and Evaluation phases. The evaluation involved expert validators specializing in Didactical Design Research (DDR), subject matter, media, and educators. The resulting Semester Lesson Plan (RPS) assessment scores and teaching materials demonstrated a "Very Good" categorization, indicating their validity and suitability for field testing. Notable findings highlight epistemological, ontogenetic, and didactic obstacles in students' understanding of quadratic inequalities. The instructional materials showed incremental effectiveness during classroom implementation, as evidenced by increasing percentages of successful sessions. This research's implications for the future include the potential adaptation of the DDR framework across diverse educational contexts, exploring technological integration, and enhancing educators' proficiency in implementing DDR-based pedagogies. By bridging theoretical foundations with practical application, this study contributes to advancing mathematics education by fostering innovative instructional materials grounded in Didactical Design Research.

INTRODUCTION
Mathematics plays an important role in everyday life because it develops everyone's way of thinking (Nugraha, 2022). Mathematics represents a crucial component of human cognition and serves as a fundamental aspect of individual capability development (Ernest, 2015). Every student must be
equipped with adequate mathematical knowledge, and classroom mathematics learning plays a vital role in this knowledge-acquisition process (Argyle, 2013). In this context, the role of the teacher becomes essential. The competence of a mathematics teacher is not merely about professional knowledge but also includes pedagogical aspects (Blömeke et al., 2020). A teacher's general competencies encompass understanding the growth of thinking and the psychological development of students, awareness of the teaching and learning process, classroom management, teaching methods, control, and evaluation (Ilanlou & Zand, 2011). According to Government Regulation No. 74/2008 article 3, a teacher must possess pedagogical, professional, personality, and social competence. A teacher's academic competence is reflected in the ability to manage learning processes, whereas professional competence is the ability to master knowledge and skills within the field of science they teach. These various aspects emphasize the importance of focusing on teacher training and competency development, especially in mathematics, to ensure that every student receives quality and relevant education tailored to their future needs.

The teaching and learning process is a complex activity (Maher et al., 2018). Shulman (1987) proposed the concept of three dimensions of professional knowledge that are important for teachers, namely Subject Matter Content Knowledge, Pedagogical Content Knowledge, and Curriculum Knowledge. Pedagogical Content Knowledge is a concept of thinking that provides an understanding that to teach science is not only enough just knowing science but also how to teach (Sukaesih et al., 2017). Some research results Shulman showed that pedagogical content knowledge is the main determinant of the success of teachers in learning teachers in learning. However, pedagogical content knowledge cannot be separated from content knowledge and pedagogical knowledge with content knowledge and pedagogic knowledge (Herold, 2019). Content knowledge pedagogic knowledge is an academic construct that describes an idea that can arouse interest in learning something (Loughran, 2005). There are three types of knowledge that knowledge that teachers must master, namely conceptual knowledge, knowledge of the structure of the material, and knowledge of specific context orientation in teaching (Gess-Newsome et al., 2019).

The results of research shows that Pedagogical Content Knowledge is the main determinant of the success of teachers in learning (Gudmunds et al., 2019) in learning (Gudmundsdottir & Shulman, 1987).

A teacher’s Pedagogical Content Knowledge (PCK) often develops in conjunction with their experience in teaching. However, the reality is that not all teachers foster experiences that contribute positively to the development of PCK (Hannula, 2017). One limiting factor may be a teacher’s habitual approach to the learning process. If the strategy remains the same for all concepts taught, and the order of presentation closely adheres to the sequence found in textbooks without adaptation to the student’s needs, this can hinder the development of PCK. It is, therefore, of vital importance to prepare prospective mathematics teachers to possess strong PCK. Since future mathematics teachers are on the path to becoming professionals, there is a continuous need to develop their professional competence to enhance the quality of education (Subanji, 2015). The utilization of Didactical Design Research-based teaching materials can provide students with better means to overcome challenges in understanding the material. These teaching materials, crafted according to the student’s learning trajectory, align with the understanding that PCK is a blend of content and pedagogical knowledge.

In conclusion, a concerted effort must be made to prepare future mathematics teachers with robust Pedagogical Content Knowledge skills. Such preparation is pivotal to improving teaching strategies, individualizing learning experiences, and ultimately elevating the educational outcomes of students. In universities, prospective mathematics teachers are trained to become professional
teachers, both in terms of mastery of material and teaching skills. The development of Didactical Design Research-based teaching materials is a methodical process structured into three distinct stages:

a. Didactical Situation Analysis Before Learning (Prospective Analysis): This stage necessitates the formulation of a Didactical Design Hypothesis, integrating an Analysis of Didactical Prerequisites (ADP). The prospective analysis lays the groundwork for understanding the prior knowledge and skills of the students, ensuring that the materials align with their learning trajectory.

b. Metapedidactical Analysis: Implementation of the prepared didactical design occurs at this phase. This multifaceted analysis encompasses a detailed examination of the teaching environment, student responses, and anticipation of reactions during the actual enactment of the didactical design. This step allows for real-time adaptation and fine-tuning of the teaching approach.

c. Retrospective Analysis: This concluding stage involves a reflective analysis that interconnects the hypothetical didactical situation’s findings with the metapedidactic analysis’s insights (Suryadi, 2010). This correlation ensures that the didactical design remains anchored in theoretical frameworks while maintaining practical relevance.

Based on this structured approach, the primary objective of the research is to conceptualize and develop mathematics teaching materials founded on Didactical Design Research. This initiative aims to augment Pedagogical Content Knowledge within the context of Three-Dimensional Geometry in upper-level Capita Selecta lectures and assess the didactical design’s quality in fostering Pedagogical Content Knowledge among prospective mathematics teachers. The quality assessment will focus on three critical parameters: validity (ensuring that the content aligns with educational standards and objectives), practicality (ensuring usability in real classroom settings), and effectiveness (ensuring that the materials achieve their intended learning outcomes). The specifications for this development process mandate a keen consideration of student responses, resulting in a highly tailored teaching resource. The creation follows a cyclical and reflective process, including:

a. Didactical Situation Analysis: Laying the groundwork by understanding student prerequisites.

b. Metapedadictic Analysis: Actively implementing and assessing the design.

c. Retrospective Analysis: Reflecting and revising to align with both theoretical understanding and practical needs. This tripartite methodology ensures that the resulting didactical design aligns coherently with the student’s learning trajectory, culminating in teaching materials that are both theoretically robust and practically applicable.

The teaching process requires understanding students’ learning obstacles, a key element in educational analysis (Suryadi, 2013). The didactic situation, which consists of the relationship between teachers, students, and materials, serves as a foundational model for analyzing and designing learning situations (Kansanen & Meri, 1999). Known as the didactic triangle, this model is a basis for analyzing learning obstacles, a critical step in preparing learning environments (Lestarai, 2019). Often, educational practitioners may overlook an analysis of why learning obstacles occur, neglecting to identify and address them adequately (Perbowo & Anjarwati, 2017). Identifying learning obstacles is not merely for modifying the current learning situation but also for devising new strategies that center on the identified epistemological barriers (Clément, 2003). Brousseau (2006) categorizes learning obstacles into three types: ontogenic obstacles, didactical obstacles, and epistemological obstacles. A thorough analysis of these obstacles aims to create an ideal didactic situation, strengthening the relationship between students and teaching materials (Carvalho et al., 2004). With this framework, the current study seeks to design teaching materials based on Didactical Design Research. These materials are oriented towards enhancing the Pedagogical Content Knowledge of prospective mathematics teachers, thereby improving pedagogical practices within the field.
METHODS

Research and Development Process with ADDIE Model

This study follows the Research and Development approach, utilizing the ADDIE model, which comprises five phases: Analysis, Design, Development, Implementation, and Evaluation (Muruganantham, 2015).

Product Development: Mathematics Teaching Materials

The outcome of this research is the creation of mathematics teaching materials based on Didactical Design Research aimed at facilitating the Pedagogical Content Knowledge of prospective mathematics teachers.

Integration of Didactical Design Research in ADDIE Model

Analysis Phase

The analysis identifies learning obstacles that stem not solely from students' abilities. Obstacles arise from students' developmental factors (ontogenical obstacles), how knowledge is formed (epistemological obstacles), and the choice of teaching materials by educators (didactical obstacles) (Brousseau, 2006). For analyzing learning obstacles in quadratic inequalities, comprehension tests are devised. Analysis of learning obstacles marks the initial step in the Didactical Design Research (DDR) process.

Design Phase

Design encompasses crafting mathematics teaching materials based on the scientific approach. Design factors in findings from the analysis phase, such as instructional needs and material structure. During this phase, research instruments are developed and validated by experts.

Development Phase

Development involves crafting mathematics teaching materials aligned with the pre-designed framework. Key steps include compiling relevant learning content, images, videos, and animations. By incorporating these elements, the aim is to enhance student engagement and interest in learning.

Implementation Phase

Testing of Developed Teaching Materials. Teaching materials are tested with students. Trial involves teaching mathematics using the scientific approach with the developed materials. Post-tests are conducted to assess students' problem-solving abilities after the learning process.

Evaluation Phase

Assessment of Developed Mathematics Teaching Materials. Evaluation considers content relevance, presentation, language, and visual design. Additionally, problem-solving tests are administered to students. These tests gauge the effectiveness of the developed teaching materials in enhancing students' mathematical problem-solving abilities.

RESULTS AND DISCUSSION

Result

The development procedure of teaching materials utilizes theories rooted in Didactical Design Research (DDR) oriented toward the Pedagogical Content Knowledge (PCK) of prospective mathematics teachers in fractional number operations. This development process employs the ADDIE
development model, which comprises five phases: Analysis, Design, Development, Implementation, and Evaluation.

a. Analysis Phase

Research studies conducted at junior high and senior high school levels have indicated that students' difficulties in learning mathematics impact their mathematical abilities (Jatisunda, 2019). Similarly, at the tertiary level, students' mathematical capabilities are significantly influenced by their ability to overcome challenges in learning mathematics (Muslim et al., 2017). These challenges manifest as obstacles experienced by students when engaging with instructional content. These obstacles arise not solely from students' abilities. Learning obstacles stem from students' developmental factors (ontogenical obstacles), factors influencing the formation of knowledge (epistemological obstacles), and obstacles arising from educators' choices of instructional materials (didactical obstacles) (Brousseau, 2006). Analysis of learning obstacles experienced by students in quadratic inequalities necessitates the creation of comprehension concept tests. These tests are structured based on the fundamental competencies of learning quadratic inequalities.

b. Design Phase

Based on findings from the analysis phase, Hypothetical Learning Trajectories (HLT) are formulated. HLT constitutes a learning process construction that anticipates student learning. It is grounded in learning objectives, students' level of understanding, and mathematical activity choices (Conner et al., 2017; Empson, 2011; Fuadiah, 2017). Learning trajectories are designed based on the previously identified learning obstacles. The function approach is selected in response to learning obstacles, determining the solution set of quadratic inequalities when given the graph of a quadratic function.

Additionally, students' sole approach (number line method) cannot solve all quadratic inequalities problems and merely promotes procedural understanding. Learning obstacles, trajectories, and the Theory of Didactical Situation (TDS) are incorporated into a didactic design for quadratic inequalities. This design encompasses three didactic methods, each containing one or more didactic situations. Situations within TDS include action, formulation, validation, and institutionalization situations (Brousseau, 2006). Didactic design is also structured based on the variety of possible learning responses. Anticipatory pedagogical didactic responses to student reactions are devised, recognizing the potential diversity in responses.

c. Development Phase

The outcome of this development phase is a draft of mathematics teaching materials based on Didactical Design Research (DDR), designed to facilitate the enhancement of Pedagogical Content Knowledge (PCK). Learning Obstacles, and Learning Trajectories, informed by the Theory of Didactical Situation (TDS), formulate a didactic design for quadratic inequalities. TDS encompasses action situations, formulation situations, and validation situations. Consequently, each case is structured based on its respective category. Beyond these three situations, the subsequent element is the institutionalization situation.

d. Implementation Phase

In the implementation phase, the drafted teaching materials undergo testing. These materials are trialed with students, involving the execution of mathematics lessons using the scientific approach and the developed materials. Furthermore, post-tests are conducted to gauge the students' problem-solving skills after learning.
e. Evaluation Phase

The evaluation phase encompasses an assessment of the developed mathematics teaching materials. This evaluation includes the suitability of the content, presentation, language, and visual design of the teaching materials. Additionally, problem-solving tests are administered to students to measure the effectiveness of the developed mathematics teaching materials in enhancing their problem-solving abilities.

The developed teaching materials underwent a thorough validation process involving experts in Didactical Design Research (DDR), subject matter, media, and educators. As depicted in the provided tables, the evaluation results consistently indicated a "Very Good" rating for both the Teaching Syllabus (RPS) and the teaching materials. It reflects the meticulous attention given to the development process, ensuring alignment with pedagogical content knowledge (PCK) goals. Notably, these assessments underscored the suitability of the teaching materials for practical application, paving the way for field testing—the field testing phase aimed to assess the practicality and effectiveness of the materials. In terms of practicality, observations of lesson implementation demonstrated a consistent improvement in the percentage of successful executions, culminating in an impressive 93% by the fifth session. Addressing feedback from validators, revisions were made to enhance the clarity and coherence of the materials, thus further refining their practicality.

Moreover, the effectiveness analysis encompassed post-implementation tests of students' pedagogical content knowledge. The results revealed an average score of 79.05, with 89.01% of students achieving the required proficiency level. These outcomes collectively validate the quality of the developed materials, confirming their suitability and effectiveness for educational settings. The rigorous validation process and the positive results from the field testing phase collectively reinforce the credibility and applicability of the teaching materials in facilitating effective mathematics education.

Discussion

Based on the findings, several learning obstacles were identified, categorized as epistemological, ontogenical, and didactical (Brousseau, 2006; Suryadi, 2013). Epistemological obstacles, such as generalization, were discovered in quadratic inequalities learning. Students struggled to generalize knowledge from equations to inequalities, mistakenly applying similar processes to equation and inequality contexts. This study revealed that mathematics education students' generalization skills were relatively low, indicating that errors weren't due to a lack of knowledge but rather the incorrect application of knowledge within the given context. Furthermore, specific epistemological obstacles emerged from this research, demonstrating that errors were attributed to the inappropriate transfer of expertise from equations to quadratic inequalities. Didactical obstacles were also noted, from instructional practices focusing solely on algebraic manipulation and the number line method without emphasizing the distinction between equations and inequalities. This limitation in didactical choices contributed to a lack of comprehensive understanding of quadratic inequalities concepts.

Prior studies have consistently highlighted the significance of understanding the role and impact of certain factors. These studies have emphasized the need to comprehensively investigate these factors to gain a deeper insight into their implications. By building upon the insights garnered from previous research, our study aims to further elucidate these factors' implications and potential applications in various contexts. By meticulously examining relevant literature and empirical data, our research seeks to contribute to the existing body of knowledge by providing a more comprehensive understanding of these crucial elements. In line with the research goals, the developed teaching materials aimed to enhance the pedagogical content knowledge of prospective mathematics teachers.
These materials comprised a Semester Lesson Plan (RPS) and instructional resources, adopting a scientific approach to strengthen problem-solving abilities. The development process followed the ADDIE model, consisting of five phases: Analysis, Design, Development, Implementation, and Evaluation. These stages ensured that the teaching materials met the valid, practical, and effective criteria. Another finding is that the development and implementation of the instructional materials based on Didactical Design Research (DDR) principles effectively addressed the identified learning obstacles. The materials successfully engaged prospective mathematics teachers in overcoming the epistemological, ontogenical, and didactical challenges associated with quadratic inequalities. Through the utilization of Hypothetical Learning Trajectories (HLT), the instructional materials provided a structured learning path that anticipated and addressed students' conceptual hurdles. This strategic approach guided learners in gradually building their understanding of quadratic inequalities and solutions, enhancing their mathematical competency. The rigorous evaluation conducted by a panel of validators, including experts in didactical design research, subject matter, media, and educators, played a crucial role in validating the effectiveness of the instructional materials. The high scores and "Very Good" ratings across various assessment aspects, such as the Course Syllabus (RPS) and instructional content, affirmed the quality and suitability of the materials for educational use.

Furthermore, implementing the materials in natural classroom settings demonstrated their practicality. The increased implementation percentage to 93% indicated that teachers could effectively utilize the materials in their instruction, contributing to a more interactive and engaging learning environment. The conclusive evidence of the materials' effectiveness came from the pedagogical content knowledge test results. With an 89.01% passing rate, the materials demonstrated their ability to enhance students' problem-solving skills in the context of quadratic inequalities. This success underscored the importance of integrating a systematic development approach with empirical testing to yield impactful educational resources. In summary, the project's findings emphasize the success of addressing learning obstacles by developing and implementing instructional materials grounded in Didactical Design Research. These materials effectively tackled conceptual challenges and proved to be practical and efficient tools for enhancing the Pedagogical Content Knowledge of prospective mathematics teachers, ultimately contributing to advancing mathematics education.

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This is an important issue for future research as it opens avenues for exploring the broader application and adaptation of the Didactical Design Research (DDR) framework in various educational contexts. Further investigation could delve into how the DDR framework could be tailored to address learning obstacles and enhance Pedagogical Content Knowledge (PCK) for other subjects beyond mathematics. Additionally, future research could delve deeper into the effectiveness and scalability of the developed instructional materials in real-world classroom settings. Longitudinal studies could provide insights into the sustained impact of the materials on students' learning outcomes over extended periods.

Furthermore, examining the potential integration of technology and digital resources within the DDR framework could be a valuable direction. It could involve exploring how digital tools can enhance the interactivity, engagement, and adaptability of instructional materials while still adhering to the principles of DDR. Exploring the professional development of educators to implement DDR-based materials effectively is also an essential avenue for future research. Investigating how teachers can be trained to understand and leverage the DDR framework could contribute to more widespread adoption and successful implementation. Overall, future research in this domain can advance our understanding of practical pedagogical approaches, contribute to developing adaptable teaching resources, and ultimately lead to improved learning experiences for students across diverse educational settings.

CONCLUSION

In conclusion, this study outlines the development of mathematics instructional materials based on Didactical Design Research (DDR) to enhance Prospective Mathematics Teachers' Pedagogical Content Knowledge (PCK) in fractional number operations. The findings indicate that the DDR approach provides a robust framework for identifying and addressing learning obstacles faced by students. The assessment results by validators and field testing indicate the successful development of the instructional materials. The Semester Lesson Plan (RPS) and the developed teaching materials received a “Very Good” rating, demonstrating their validity, practicality, and effectiveness in supporting mathematics education. Noteworthy findings encompass epistemological, ontogenetic, and didactic obstacles in quadratic inequalities learning. Evaluation outcomes show an increment in the implementation of the learning process as the sessions progress, reaching a significantly high level by the fifth session. The significance of these findings for future research endeavors is underscored. Subsequent studies may explore the application and adaptation of the DDR framework across diverse educational contexts and delve into the effectiveness of the developed instructional materials in real classroom settings. Integrating technology within the DDR framework and enhancing educators' professionalism in materializing DDR-based pedagogy are pertinent areas for further investigation. Consequently, this study contributes to advancing innovative instructional tools supported by the Didactical Design Research theory, paving the way for enhanced mathematics education quality and content-based instruction in the foreseeable future.
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