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Mapping Geometric Minds: Exploring 3D Thinking Skills of Elementary School Students Using the Van Hiele Model

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Abstract
This study investigates the three-dimensional (3D) geometric thinking skills of elementary school students based on the Van Hiele Model. Using a grounded theory approach, the research was conducted in elementary schools across West Java and Central Java provinces in Indonesia. Data were collected through tests, observations, and interviews, focusing on students' abilities at the visualization, analysis, and abstraction levels of geometric thinking. Results indicate that students generally perform well at the visualization level, with 77.14% of Majalengka students and 71.43% of Surakarta students achieving this level. At the analysis level, 62.86% of Majalengka students and 67.86% of Surakarta students demonstrated proficiency. The abstraction level posed the greatest challenge, with only 48.57% of Majalengka students and 53.57% of Surakarta students reaching this level. The study reveals that while students have a good grasp of basic 3D geometric concepts, they struggle with more abstract thinking and efficient problem-solving methods. These findings highlight the need for strengthened instruction at the visualization and analysis levels, as well as increased emphasis on developing abstract thinking skills in geometry education. The research underscores the effectiveness of the Van Hiele Model in assessing geometric thinking development and provides valuable insights for curriculum development and teaching strategies in elementary geometry education. Further research is recommended to explore the understanding of more complex geometric shapes and to employ more in-depth observational methods.

INTRODUCTION

At the elementary education level, the introduction of mathematical concepts aims to build a comprehensive understanding foundation for students. This is done to minimize misunderstandings of basic mathematical concepts. However, many elementary school students still struggle to grasp these concepts. This difficulty is exacerbated by the abstract nature of mathematics, which is often hard for students to comprehend. According to Piaget & Inhelder (1969), children of elementary school age are in the concrete operational stage and cannot yet think formally. Mathematics, as a deductive science with symbolic language (Rott 2021), adds to the complexity of understanding the material. At the basic level, mathematics is often presented in abstract forms that contradict students' concrete thinking patterns, which are more accustomed to real-life experiences.

One of the mathematical materials taught at the elementary level is geometry, an integral part of the mathematics curriculum (Silva et al. 2015). Geometry teaching in elementary schools is generally conducted using two-dimensional images. However, this approach is often ineffective in teaching the concepts of three-dimensional (3D) geometric objects (Olkun 2003; Battista 2003). Representing 3D images in two dimensions is often insufficient to help students understand the concepts and properties of 3D objects (Ocal & Halmatov 2021; İbili et al. 2020). Properties of 3D objects, such as edge length, the number of sides, and relationships between elements like edges and angles, are complex structures that are difficult for students to grasp (Battista 2007). Pavlovičová and Švecová (2015) reported that students often struggle to find elements forming the same side in the net of 3D objects and understand their properties.

To address the problems in 3D geometry learning, the first step is to research the 3D geometric thinking skills of elementary students. The main issue in this research is how the 3D geometric thinking skills of elementary students are, considering the complexity of the material and the constraints in representing images of 3D objects. Understanding students' comprehension levels of 3D geometry concepts is crucial for designing more effective learning approaches that support the development of students' mathematical understanding at the basic level.

Geometry is one of the branches of mathematics considered difficult and feared by students (Adolphus 2011). However, it is also important to consider the level of geometric thinking students achieve. Geometric thinking skills can help students develop critical thinking skills (Hassan et al. 2020). These skills have been researched for years by many experts (Ismail & Rahman 2017; Abdullah & Zakaria 2013; Armah & Kissi 2019; Meng & Sam 2013; Haviger & Vojkůvková 2015; Siew et al. 2013; Abu & Abidin 2013). The Van Hiele model of geometric thinking is appropriate for identifying students' geometric thinking skills (Crowley 1987; Usiskin 1982; Clements & Battista 1992; Naufal et al. 2021).

This research uses the Van Hiele theory to assess students' 3D geometric thinking levels. The Van Hiele theory divides geometric thinking into five levels: (1) Visualization, (2) Analysis, (3) Abstraction deduction, (4) Deduction, and (5) Rigor. Each of these levels has characteristics that correspond to students' geometric thinking processes (Haviger & Vojkůvková 2014; Gilar Jatisunda & Nahdi 2020; Alex 2019). These levels must be passed sequentially without skipping any (Abu et al. 2012). Many researchers worldwide apply this model to observe students' geometric thinking levels (Ding & Jones 2006; Idris 1999; Usiskin 1982; Wu & Ma 2005). This theory shows that students go through various levels of geometric thinking, from recognizing shapes to building formal geometric proofs (Abu et al. 2012; Pierre M Van Hiele 1999). The Van Hiele theory explains why many students have difficulty in geometry and offers a teaching model to improve students' geometric thinking levels (Fuys et al. 1988; P. M. Van Hiele 1986).

Several studies on the Van Hiele geometric thinking model show that understanding students' geometric thinking skills can help teachers determine solutions to geometry learning problems, thus achieving learning objectives. The Van Hiele model helps teachers design learning strategies to enhance students' geometric thinking skills. However, most geometry studies have focused on middle school students' thinking skills and 2D thinking skills, while research on 3D geometric skills in elementary students is still limited. Therefore, this study aims to examine elementary students' 3D geometric thinking skills based on the Van Hiele model.

METHODS

This research aims to describe the 3D geometric thinking skills of elementary students. This study uses the grounded theory method (Corbin & Strauss 1990), a series of procedures designed to build a theory regarding 3D geometric thinking skills. The research was conducted on students from several elementary schools in two provinces, West Java and Central Java. Adopting a qualitative approach, this research attempts to interpret the meaning of the collected data and understand social problems based on participants or other data sources.

Data collection was carried out through tests, observations, and interviews. Tests were conducted to determine students' three-dimensional geometric thinking skills through their answers. Written tests consisted of five questions on flat-sided solid shapes based on Van Hiele's theory, covering visualization, analysis, and abstraction levels. Observations were conducted to see the stages of students' three-dimensional geometric thinking when answering geometry questions. Interviews were conducted to address issues found from the test and observation results to validate the findings.

Data analysis in this study was divided into three stages: analysis before, during, and after fieldwork. Before fieldwork, analysis was conducted on preliminary study results or secondary data to determine the research focus. During data collection in the field, analysis was done using the interactive model of Miles and Huberman, which is continuous until the data is saturated. Activities in this model include data reduction, data presentation, and conclusion drawing and verification.

RESULTS AND DISCUSSION

Results

Description of Test Results

This study aims to assess elementary school students' 3D geometric thinking skills based on the Van Hiele Model. Six test questions, aligned with the geometric thinking levels of the Van Hiele Theory, were administered. The summary of the students' geometric thinking test results from elementary schools in Majalengka and Surakarta is presented in Table 1.

Tabel 1. Students' Geometric Thinking Test Results Based on the Van Hiele Model

No.	Elementary School	Levels of Geometric Thinking Achieved		
		Visualisasi	Analysis	Abstraction
1	Majalengka	77,14%	62,86%	48,57%
2	Surakarta	71,43%	67,86%	53,57%

Based on Table 1, the test results of students' geometric thinking abilities according to the Van Hiele Model reveal interesting differences between students from elementary schools in Majalengka and Surakarta.

Students from elementary schools in Majalengka demonstrated strong visualization skills, with 77.14% of the students able to recognize and describe 3D geometric shapes. This indicates that the majority of students at this school have a good basic understanding of the forms and structures of solid figures.

However, when it comes to the analysis level, only 62.86% of the students were able to achieve this level. This suggests that while more than half of the students can analyze and distinguish the elements of solid figures in more detail, there is still room for improvement. At the abstraction level, which requires more abstract thinking, only 48.57% of the students were able to reach this level. This indicates that nearly half of the students are beginning to understand geometric concepts more abstractly, although not fully.

On the other hand, students from elementary schools in Surakarta showed slightly different results. At the visualization level, 71.43% of the students were able to achieve this level, slightly lower than the students from Majalengka. This shows that the majority of students in Surakarta also have the basic ability to recognize and describe geometric shapes, although slightly fewer compared to the students in Majalengka.

However, at the analysis level, students in Surakarta showed an advantage, with 67.86% of the students able to achieve this level. This indicates that the majority of students in Surakarta can analyze and distinguish the elements of solid figures more deeply. At the abstraction level, 53.57% of the students from Surakarta were able to achieve this level, which is also higher compared to the students from Majalengka. This shows that more than half of the students in Surakarta are beginning to think abstractly about the properties of geometry.

Overall, although both schools demonstrated good skills at various levels of geometric thinking, Surakarta slightly outperformed in analytical and abstract thinking skills, while Majalengka showed strength in basic visualization skills. These results provide valuable insights into the geometric thinking levels of students in both schools and highlight areas that need improvement to achieve a more comprehensive understanding of 3D geometry.

Analysis of Student Responses

From the students' responses, the following are examples of student answers to the geometric thinking test questions based on the Van Hiele Theory.

1. Visualization Level

The visualization level of the Van Hiele model can be identified from the students' processes in answering questions 2 and 3. An example of a student’s answer to question number 2 can be seen in Figure 1.

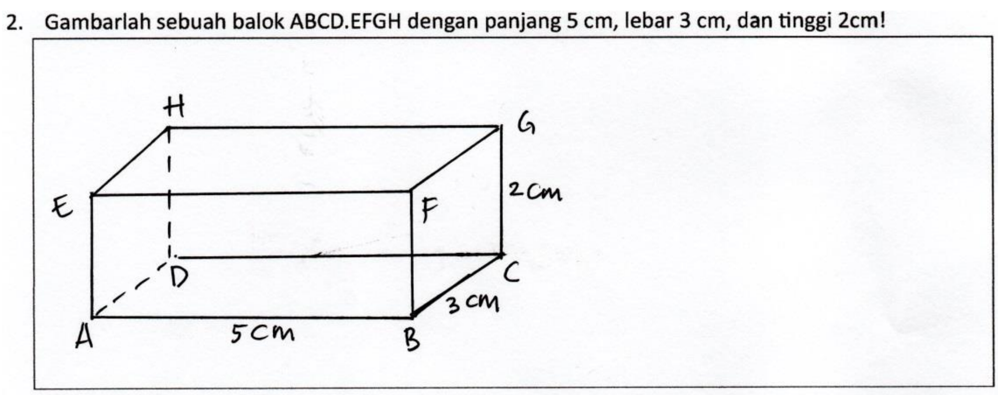
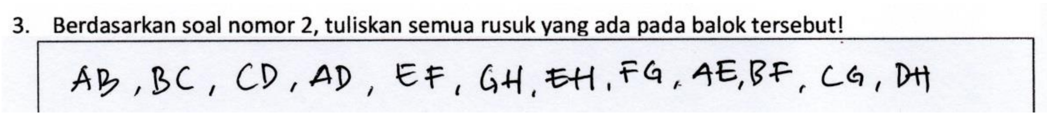


Figure 1. Student's answer to question number 2 (visualization level)

In question number 2, students were asked to draw a rectangular prism ABCD.EFGH with edge lengths of 5 cm, 3 cm, and 2 cm. From Figure 1, it is evident that the student's answer indicates an understanding of the question's requirements, demonstrating the visualization level. The student at this level can comprehend the task of drawing a rectangular prism with the given edge lengths of 5 cm, 3 cm, and 2 cm. Essentially, the student understands and can accurately draw the rectangular prism. This is further corroborated by interview results, which show that the student could identify the rectangular prism with the specified edge lengths as requested in the question. Based on the test results from several students and supported by interview findings, it can be concluded that, in general, students can solve spatial questions based on the Van Hiele Theory and reach the visualization level. This indicates that students at this level have a basic understanding of the overall shape and structure of 3D figures without analyzing the properties of their individual components in depth.


3. **Berdasarkan soal nomor 2, tuliskan semua rusuk yang ada pada balok tersebut!**

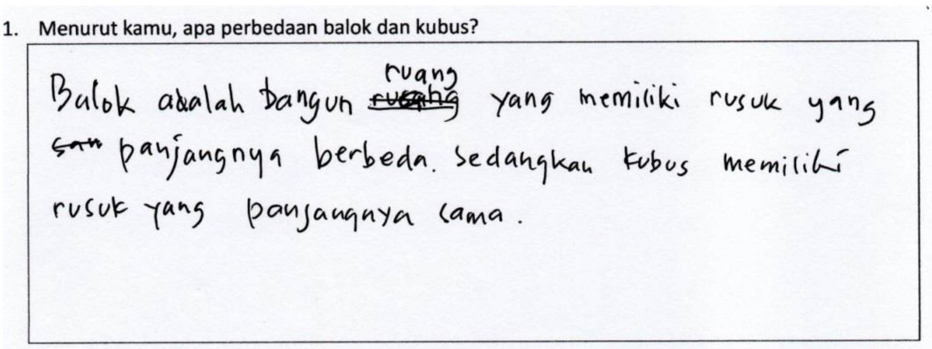
AB, BC, CD, AD, EF, GH, EH, FG, AE, BF, CG, DH

Figure 2. Student's answer to question number 3 (visualization level)

In question number 3, students were asked to name the edges of the rectangular prism described in question number 2. From the answer shown in Figure 2, it is clear that the student understands the question's requirements, demonstrating the visualization stage. The student at this level can correctly name all the edges of the rectangular prism. During the interview, the student was also able to identify each edge of the prism. This means the student has reached the visualization level. Based on the test results from several students and supported by interview findings, it can be concluded that, in general, students can solve spatial questions based on the Van Hiele Theory and reach the visualization level.

2. Analysis Level

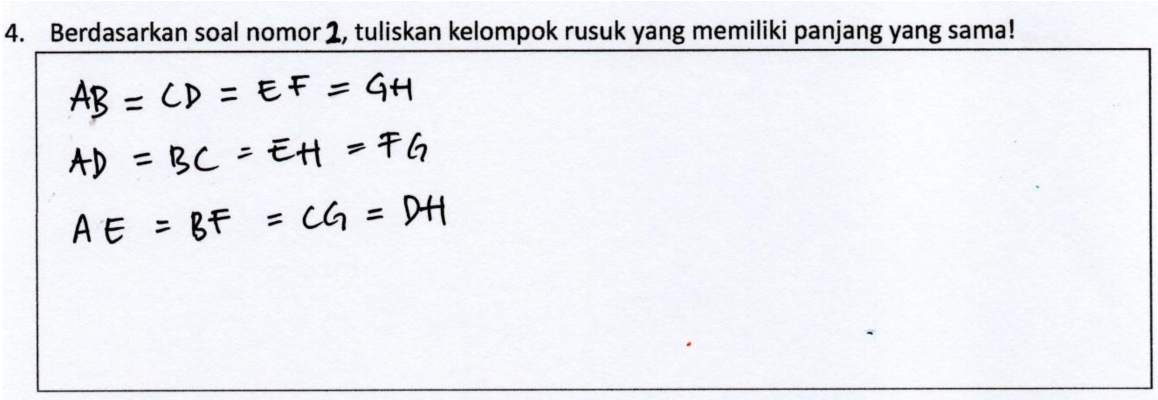
The analysis level of geometric thinking in the Van Hiele model can be identified from the students' processes in answering questions 1 and 4 given by the researcher.


1. **Menurut kamu, apa perbedaan balok dan kubus?**

Balok adalah bangun ~~ruang~~ ruang yang memiliki rusuk yang ~~sam~~ panjangnya berbeda. Sedangkan kubus memiliki rusuk yang panjangnya sama.

Figure 3. Student's answer to question number 1 (analysis level)

In question number 1, students were asked to explain the differences between a rectangular prism and a cube. Based on the student’s answer shown in Figure 3, it is clear that the student understands the question's requirements, demonstrating the analysis level. The student at this level can differentiate and compare the two geometric figures, a cube and a rectangular prism. Most students were able to answer the similarities and differences between cubes and rectangular prisms based on their properties. The students explained that both shapes are 3D figures with 6 faces, 12 edges, and 8 vertices. They also described the differences, noting that a rectangular prism has edges of different lengths with opposite faces being equal, while a cube has faces of equal size. Interviews also revealed that students could identify the elements and properties of both shapes, indicating that they have reached the analysis level.


4. Berdasarkan soal nomor 2, tuliskan kelompok rusuk yang memiliki panjang yang sama!

AB = CD = EF = GH

AD = BC = EH = FG

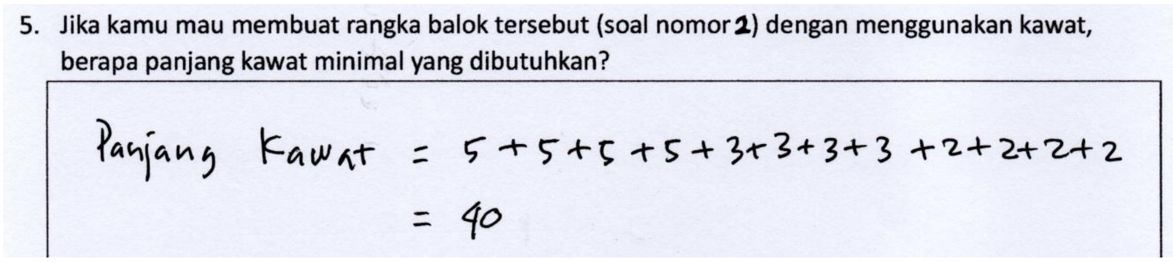
AE = BF = CG = DH

Figure 4. Student's answer to question number 4 (analysis level)

In question number 4, students were asked to group the edges of the rectangular prism described in question number 2 based on their lengths. From the student's answer shown in Figure 4, it is clear that the student understands the question's requirements, demonstrating the analysis stage. The student at this level can group the edges of the rectangular prism according to their lengths. This indicates that the student has reached the analysis level.

3. Abstraction Level

The abstraction level of the Van Hiele model can be identified from the students' processes in answering question number 5.


5. Jika kamu mau membuat rangka balok tersebut (soal nomor 2) dengan menggunakan kawat, berapa panjang kawat minimal yang dibutuhkan?

Panjang Kawat = 5 + 5 + 5 + 5 + 3 + 3 + 3 + 3 + 2 + 2 + 2 + 2

= 40

Figure 5. Student's answer to question number 5 (abstraction level)

In question number 5, students were asked how much minimum wire length is needed to make a rectangular prism frame, related to question number 2. The test results for question number 5 show that most students were able to determine the wire length, although many used less systematic approaches. A few students succeeded in using a more efficient method by multiplying the sum of the length, width, and height by four, following the systematic steps expected. Interviews revealed that while students understood the concept of edges, they did not fully grasp the systematic approach required. This indicates that students are in the early stages of achieving the abstraction level, where they have the conceptual understanding but have not yet fully applied it in the most efficient manner.

Discussion

This study aims to explore elementary school students' 3D geometric thinking skills based on the Van Hiele Model. The research involved administering six test items, developed according to the geometric thinking levels in the Van Hiele Theory, to students from two different elementary schools:

one in Majalengka and the other in Surakarta. The test data were then analyzed to determine the extent to which students could reach the visualization, analysis, and abstraction levels in geometric thinking.

The test results revealed interesting variations between the students from the two schools. In the Majalengka elementary school, 77.14% of students were able to reach the visualization level, where students can recognize and depict 3D geometric shapes at a basic level. This indicates that the majority of students at this school have a good basic understanding of the shapes and structures of geometric figures. However, only 62.86% of students managed to reach the analysis level, which requires them to analyze and differentiate the elements of geometric figures in more detail. At the abstraction level, which requires more abstract thinking, only 48.57% of students reached this level. These data suggest that while most students have begun to understand more abstract geometric concepts, there is still significant room for improvement.

Conversely, students from the Surakarta elementary school exhibited a slightly different pattern. At the visualization level, 71.43% of students reached this level, which is slightly lower than the students from the Majalengka school. However, at the analysis level, 67.86% of students from the Surakarta school achieved this level, indicating that a majority of students at this school have better abilities in analyzing and differentiating the elements of geometric figures. At the abstraction level, 53.57% of students from the Surakarta school reached this level, showing that more than half of the students are beginning to think abstractly about geometric properties.

Overall, although both schools demonstrated good capabilities at various levels of geometric thinking, the Surakarta school showed a slight advantage in analysis and abstract thinking abilities, while the Majalengka school excelled in basic visualization skills. These findings provide valuable insights into the levels of geometric thinking skills among students in both schools and highlight areas that need improvement to achieve a more comprehensive understanding of 3D geometry.

In the analysis of student answers to the test questions, at the visualization level, which is the most basic level in the Van Hiele theory, students showed fairly good abilities. This was evident from their answers to questions 2 and 3. In question number 2, students were asked to draw a rectangular prism ABCD.EFGH with specified edge lengths. The results showed that students could understand the instructions and draw the prism according to the required dimensions. This ability indicates that students can visualize 3D geometric shapes well. Subsequently, in question number 3, students were asked to name the edges of the rectangular prism. The students' answers demonstrated a good understanding of the edge concept, an important indicator at the visualization level. Interview results also supported this finding, as students could correctly identify each edge of the prism. These findings indicate that most students have reached the visualization level in their 3D geometry understanding. According to Cesaria (2021), at this level, students recognize geometric shapes solely as the visual characteristics of an object. The visualization level refers to the students' ability to recognize shapes and their names (Baiduri et al., 2022).

Moving to a higher level, the analysis level, this study used questions 1 and 4 to measure students' abilities. At this level, students are expected to analyze the properties of geometric shapes and make simple generalizations. Question number 1 asked students to explain the differences between a rectangular prism and a cube. The results showed that students could differentiate and compare the two geometric shapes based on their elements. They could explain the similarities and differences quite well, demonstrating a deeper understanding of the geometric properties of both shapes. In question number 4, students were asked to group the edges of the rectangular prism based on their lengths. Students' ability to answer this question correctly shows that they have reached the analysis level, where they can analyze and group geometric elements based on their properties. Thus, the students have already achieved the analysis stage. According to Baiduri et al. (2022), at the analysis stage, students can recognize images based on their characteristics, analyze, and name the properties of the images.

The highest level measured in this study is the abstraction level, tested through question number 5. At this level, students are expected to make connections between geometric properties and use abstract deduction. This question asked students to calculate the minimum wire length needed to construct a rectangular prism frame. The results showed that most students could identify the edge concept in the prism and calculate the required wire length. Interestingly, most students solved this question using a simpler method, summing all the edges of the prism, rather than the more systematic method expected. Only a few students used a more efficient method by multiplying four by the sum of the length, width, and height of the prism. According to Supli & Yan (2024), students have the ability to apply their knowledge of shapes and figures to a wider range of situations during this period. They begin to recognize patterns in shapes and use these patterns to predict the characteristics of new shapes. Additionally, they start to understand how shape properties are interconnected (Stols et al., 2015). These findings indicate that while students have reached the abstraction level in some aspects, there is still room for development in terms of efficiency and systematic geometric thinking.

Further analysis of these research results shows that the elementary school students studied have fairly good 3D geometric thinking skills, especially at the visualization and analysis levels. They can recognize and depict geometric figures, identify their elements, and compare and analyze their geometric properties. However, when dealing with more abstract concepts requiring more systematic thinking, as seen at the abstraction level, students tend to use simpler, more direct methods. According to Patkin (2011), students' difficulties in understanding mathematical concepts are due to the many concepts that have different meanings in everyday life and various mathematical terms in different contexts. Some studies mention that many students at all levels experience misconceptions about geometric concepts (Marchis, 2012; Utami et al., 2017) as well as difficulties among elementary and middle school students, and even adults, in visualizing 2D-3D dimensions and vice versa (Hershkowitz, 1990; Barkai & Patkin, 2012). For example, when they see a ball in reality, it will appear as a circle in a picture because it is difficult to depict 3D in a 2D image. This is due to the teaching methods used in geometry instruction (Sam & Yong, 2007), teachers neglecting spatial relationships (Karakuş & Peker, 2015), and ineffective textbooks (Hershkowitz, 1987).

These findings have important implications for the teaching of geometry at the elementary school level. First, it is crucial to maintain and strengthen instruction at the visualization and analysis levels, as these form a strong foundation for further geometric understanding. Second, there needs to be greater emphasis on developing abstract and systematic thinking skills in geometry. Teachers may need to design learning activities that gradually guide students from simple to more efficient and abstract problem-solving methods. Additionally, this study demonstrates the effectiveness of the Van Hiele Model in assessing and understanding students' geometric thinking development. The model provides a clear framework for identifying students' understanding levels and can assist teachers in designing instruction that aligns with students' cognitive development levels. The Van Hiele theory is useful in analyzing learners' performance (Alex & Mammen, 2012). It has had a significant impact worldwide in terms of geometry education, especially after its influence on Russian mathematics education became internationally recognized (Martin, 2007; Stols et al., 2015).

However, it should be noted that this study has several limitations. First, it focuses on polyhedral shapes, specifically cubes and rectangular prisms. Further research may be needed to assess students' understanding of more complex geometric shapes. Second, although this study used interviews to support the findings, it might be beneficial to conduct direct observations of students' thinking processes while solving geometric problems. Overall, this study provides valuable insights into elementary school students' 3D geometric thinking skills. These findings can serve as a basis for developing more effective geometry curricula and teaching strategies at the elementary school level, with the ultimate goal of enhancing students' overall geometric understanding and skills.

CONCLUSION

The results of the study indicate that students from both schools have fairly good abilities at the visualization level, with the ability to recognize and depict 3D geometric shapes. Majalengka Elementary School showed slightly better performance at this level, with 77.14% of students reaching this level, compared to 71.43% in Surakarta Elementary School. At the analysis level, where students are expected to differentiate and compare geometric shapes based on their properties, Surakarta Elementary School showed an advantage with 67.86% of students reaching this level, compared to 62.86% in Majalengka Elementary School. The abstraction level proved to be the biggest challenge for students from both schools, with only 48.57% of Majalengka Elementary School students and 53.57% of Surakarta Elementary School students able to reach this level. At this level, students tended to use simpler rather than systematic problem-solving methods when dealing with more abstract concepts.

These findings have important implications for the teaching of geometry at the elementary school level. First, it is crucial to maintain and strengthen teaching at the visualization and analysis levels, as these provide a strong foundation for further geometric understanding. Second, there needs to be greater emphasis on developing abstract and systematic thinking skills in geometry. Teachers may need to design learning activities that gradually guide students from simple to more efficient and abstract problem-solving methods.

This study also demonstrates the effectiveness of the Van Hiele Model in assessing and understanding students' geometric thinking development. The model provides a clear framework for identifying students' understanding levels and can assist teachers in designing instruction that aligns with students' cognitive development levels.

While this study provides valuable insights, it should be noted that there are some limitations. The study focuses on polyhedral shapes, specifically cubes and rectangular prisms, so further research may be needed to assess students' understanding of more complex geometric shapes. Additionally, although this study used interviews to support the findings, direct observation of students' thinking processes while solving geometric problems might provide deeper insights.

Overall, this study provides a strong foundation for developing more effective geometry curricula and teaching strategies at the elementary school level. By understanding the patterns of 3D geometric thinking skills among students, educators can design more targeted and effective learning, with the ultimate goal of enhancing students' overall geometric understanding and skills. Further research with broader coverage and more in-depth observational methods will be highly beneficial in deepening our understanding of the development of elementary school students' geometric skills.

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