How Elementary Students Communicate Their Mathematical Problem Solving in Writing

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Abstract
Mathematics plays a crucial role in education, particularly in the development of problem-solving abilities. The proficiency in mathematical problem-solving has garnered significant attention, especially in light of the low academic performance of Indonesian students in the 2018 PISA assessment. This research aims to analyze the mathematical problem-solving skills of elementary school students using the Polya Method. The study employs a qualitative research design with a phenomenological approach. Subjects were selected through purposive sampling, representing three ability groups. Data were collected through tests, interviews, and observations, and were subsequently analyzed using the qualitative approach of Miles & Huberman. The findings indicate High Category Mathematical Problem-Solving Ability (HPSA) exhibit excellent mathematical problem-solving skills. They can correctly answer all questions, comprehend problems, plan, execute, and review their answers thoroughly. Moderate-ability students (MAT) demonstrate relatively good skills, although they tend to rush through problem-solving and overlook the importance of double-checking. Low-ability students (LAT) encounter difficulties in understanding problems, planning, and implementing solutions. They exhibit less systematic approaches and frequently make errors. The discussion concludes that the understanding of problems, planning, and the implementation of Polya's problem-solving steps need improvement, especially among moderate and low-ability students. Teachers should provide more intensive guidance, particularly in understanding problems and planning solutions. This research provides a profile of students' problem-solving skills and serves as a foundation for the development of more effective mathematics learning strategies in schools.

INTRODUCTION
Mathematics is a discipline that plays a significant role in human life, especially in the context of education. The primary goal of mathematics learning is to develop students' abilities in high-level thinking and shape positive attitudes, particularly through critical and creative thinking. Students are expected to exhibit a resilient, diligent, and confident attitude when facing unfamiliar situations. Achieving these goals can be realized through learning mathematical problem-solving.

Problem-solving skills play a central role in the mathematics learning process within the education system. According to Zhou et al. (2020), mathematics is the core subject most closely related to problem-solving. National curriculum standards in various countries, such as Australia, Turkey, the United Kingdom, Canada, India, Ireland, Scotland, Singapore, South Africa, Sweden, and the United States, emphasize the importance of competencies such as problem-solving, reasoning, and the ability to connect mathematical concepts (Boesen et al., 2014; Davis et al., 2014; Prendergast et al., 2018). The National Council of Teachers of Mathematics
(NCTM, 2000) states that problem-solving cannot be separated from mathematics learning and is an integral part of the process. Roebayanto & Harmini (2017) emphasize that problem-solving is a crucial component in the mathematics curriculum, enabling students to apply their knowledge and skills in solving non-routine problems. The focus on problem-solving in mathematics learning aims to enhance students' abilities to understand, formulate mathematical models, solve, and interpret problem solutions (Herlawan & Hadija, 2017; Nugraha, 2022). Problem-solving abilities involve students' efforts to solve problems by utilizing their knowledge and skills. This not only involves the use of methods taught by teachers but is more about the process of elaborating students' abilities, where students can find combinations of learned rules to develop new approaches (Aisyah et al., 2018; Wawat, 2022). Problem-solving also includes the application of previous knowledge and skills possessed by students, in line with the concept expressed by Novitasari & Wilujeng (2018), stating that problem-solving ability is the skill to find solutions to a condition with an unknown solution directly, by combining concepts and rules of mathematics understood previously.

Students' ability to solve mathematical problems is an important aspect that requires serious attention, especially considering the low achievement in mathematical problem-solving in Indonesia, as revealed in the results of the Programme for International Student Assessment (PISA) 2018 study. The data indicates that Indonesian students are ranked 74th out of 79 participating countries, with a score of 379 (OECD, 2018). Research by Tomo et al. (2016) asserts that students' low problem-solving abilities are due to a lack of mastery of material and thinking skills. Further research, as conducted by Anugraheni (2019), shows that the Polya method can be effective in improving students' mathematical problem-solving abilities. In the past decade, research examining students' abilities to solve mathematical problems using the Polya method has been extensively conducted. Lee (2016) in his study indicated that the majority of students utilizing the Polya method were fairly content with the proposed problem-solving steps. Thiangthung's research (2016) reported that based on assessments, exercises, worksheets, homework, and tests, students were able to enhance their abilities to solve mathematical problems. Similarly, Yapatang & Polyiem's study (2022) also demonstrated that the management of learning designed through cooperative learning application and the Polya problem-solving process effectively developed students' mathematical problem-solving abilities. Based on these research studies, the researcher is intrigued to investigate students' proficiency in solving mathematical problems. However, distinct from these previous studies, this research will focus on the implementation of the Polya Method as a strategy to enhance the mathematical problem-solving skills of elementary school students.

The Polya Method, proposed by Polya (1973), offers four essential steps to solve problems: understanding the problem, planning the problem-solving, implementing the problem-solving plan, and reviewing the completeness of the problem-solving. By applying Polya's theory, it is expected that students can develop a better understanding of mathematical problems, identify known, unknown, and required elements, and plan and implement solutions more effectively.

The analysis of students' mathematical problem-solving abilities using the Polya method is expected to provide a profile of students' abilities at high, medium, and low levels. The results of this analysis can serve as a basis for the development of better learning strategies to improve the quality of mathematical problem-solving in schools. Therefore, the aim of this research is to analyze elementary school students' mathematical problem-solving abilities by applying the Polya method, focusing on understanding and implementing problem-solving steps, and evaluating the profile of students' abilities at high, medium, and low levels.

**METHODS**

This research adopts a qualitative method with a phenomenological approach to explore and describe the common meaning given by individuals to their life experiences related to the phenomenon of mathematical problem-solving. The research location focuses on the fifth-grade class
of SD 2 Dongos, Kedung District, Jepara Regency. Research subjects were selected through purposive sampling, a sample determination technique that considers specific characteristics (Lestari & Yudhanegara, 2017). Six students were chosen as samples, representing three ability groups: high, medium, and low, with each group having one research subject. The purpose of this sampling was to analyze the use of Polya's problem-solving steps in students.

Data were collected through interviews, observations, and documentation. Primary data were obtained from the results of mathematical problem-solving ability tests, interviews, and direct observations of students. Meanwhile, secondary data included documents of students' work results in mathematical problem-solving using the Polya method, as well as photos of interview and observation results. Data analysis was conducted qualitatively, following the Miles & Huberman (1994) approach involving data reduction, data presentation, and drawing conclusions.

To ensure the validity and reliability of data in qualitative research, the researcher used triangulation techniques. Triangulation was performed by comparing the results of interviews conducted by the researcher, ensuring that the obtained data is reliable and provides a more holistic overview of students' experiences in mathematical problem-solving using the Polya method.

RESULTS AND DISCUSSION

Result

The data collected in this study pertain to students' abilities in solving mathematical problems. Students were given non-routine problems related to the volume of three-dimensional figures. The following is an example of the given problem:

"A rectangular bathtub has a length of 40 cm, a width of 60 cm, and a height of 80 cm. The bathtub will be filled using a cube-shaped ladle with a side length of 40 cm. How many ladles are needed to fill the bathtub to the brim?"

After completing the test, interviews were conducted with students to obtain a concrete understanding of their problem-solving abilities related to the volume of cubes and rectangular prisms. Data were collected through documentation of test results, including students' answer sheets, and using interview guidelines.

Upon finding the problem-solving test and conducting interviews with selected subjects, the data obtained were analyzed to assess students' mathematical problem-solving abilities based on the formulated problem. The following is a presentation of data and analysis of problem-solving abilities for each category:

**High Category Mathematical Problem-Solving Ability (HPSA)**

In answering non-routine problems, the High Category Mathematical Problem-Solving Ability (HPSA) attempted to understand the information in the problem statement (see Figure 1). After comprehending it, the HPSA could articulate what was mentioned in the problem statement. The HPSA was able to complete the problem-solving indicators effectively. The HPSA could fully explain what was known and stated in the problem, demonstrating a systematic problem-solving approach. However, in analyzing the problem, the subject did not specify each unit for every numerical element in the problem. Nevertheless, during the interview regarding the completed work, the subject did not take full responsibility for the results.
In answering the given problem, the Moderate Category Mathematical Problem-Solving Ability (MPSA) understood the information provided, but the MPSA did not meet the indicators well (see Figure 2). The MPSA could write down what was known and derived from the problem, but the MPSA made a mistake by not explicitly stating "given" and "asked." The MPSA hurriedly wrote down the numbers present in the problem without using the specified terms. The subject acknowledged their shortcomings during the interview related to the completed work. As supporting data for the conducted test, the researcher interviewed the student.
Based on the interview results, the answers from the MPSA did not meet the indicator of understanding the problem because the MPSA could not write down what was known and what was asked. The subject did not write according to the problem-solving indicator. The interview results from the MPSA were consistent with what was written on the answer sheet. The following is an explanation of the results from the moderate category subject according to Polya's stage indicators.

1. **Understanding the Problem:**
   In the understanding the problem stage, the moderate-category subject was not able to comprehend the problem well. However, the subject could write down the known and asked elements, finding the question correctly, although the wording was unclear, lacking the terms "given" and "asked." The subject's lack of precision led to many errors in writing the mathematical model in the subject's response.

2. **Devising a Plan:**
   In the problem-planning stage, the moderate-category subject could make a plan but could not write down the problem-solving plan on the answer sheet. However, during the interview, the subject could explain the formula to be used to answer the question. The subject could analyze the information accurately. Therefore, the moderate-category subject is said not to have exceeded the maximum planning stage indicator.

3. **Carrying out the Plan:**
   At this stage, the moderate-category subject could present the solution steps correctly, and the calculations were accurate. In the process, the subject had executed well according to the chosen steps and produced the correct answer. Thus, the moderate-category subject exceeded the indicator of carrying out the problem-solving plan. This subject could solve the problem without having to write down the solution plan first. The subject did not adhere to Polya's steps, but the subject could execute and solve the problem correctly.

4. **Looking Back:**
   At the looking back stage, the moderate-category subject did not check back on each step that had been worked on. This was evidenced by the subject not writing conclusions for each problem worked on because the subject was confident in the results, so there was no need to double-check the work. Thus, the moderate-category subject could only exceed three indicators in solving problems according to Polya, namely understanding the problem, planning the problem-solving, and carrying out the problem-solving plan.
**Low Category Mathematical Problem-Solving Ability (LPSA)**

From the given answers, the Low Category Mathematical Problem-Solving Ability (LPSA) did not understand the indicator of understanding the problem in solving mathematical problem-solving (see Figure 3). The LPSA understood the problem in their own language, and their understanding was still not accurate. As supporting data for the conducted test, the researcher interviewed the student. Based on the interview results, it was found that the LPSA could not meet the problem-solving indicator of understanding the problem accurately.

![Image of LPSA answers]

**Figure 2. The answers of Moderate Category Mathematical Problem-Solving Ability**

The LPSA could not write down what was known and what was asked. The interview results from the LPSA were consistent with what was written on the answer sheet. The following is an explanation of the results from the low category subject for each indicator according to Polya’s stages.

1. **Understanding the Problem:**
   According to the LPSA’s answer sheet, the subject could reach the stage of understanding the problem based on Polya, but not entirely accurately and correctly in their writing. The subject seemed to struggle in understanding the problem. The subject could identify elements contained in the problem, but there were many errors in translating them into a mathematical model. This proves that low-category subjects need guidance in understanding story problems or problem-solving related to daily life.

2. **Devising a Plan:**
   In the stage of devising a problem-solving plan, the low-category subject had written down the problem-solving plan on their answer sheet, but the chosen plan was not appropriate. Similarly, during the interview, the subject could not explain the meaning of the problem. The subject struggled to analyze story problems, so they could not perform calculations in the next stage. The subject had difficulty remembering and determining formulas related to square and rectangular shapes. The students' difficulty in solving story problems is their difficulty in understanding the story. Thus, the subject is said not to have succeeded in exceeding the problem-solving indicator at the planning stage.

3. **Carrying out the Plan:**
   At the stage of carrying out the problem-solving plan, the low-category subject could not solve the problem because they could not choose the correct strategy and had difficulty performing calculations, resulting in an incorrect answer. The subject had difficulty with calculations, so they could not meet the indicator at the stage of carrying out the plan. Consistent with the research by Putra et al. (2018), the results showed that students' mathematical problem-solving was still low. Thus, the low-category subject could not meet the indicator at the stage of carrying out the problem-solving plan.

4. **Looking Back:**
   In the final stage of looking back at the answers, the low-category subject did not double-check because they could not solve the problem. From the stage of selecting a strategy to implementing the strategy, the subject could not complete it. Thus, the subject could not exceed the looking back stage in the Polya stages.
Discussion

High Category Mathematical Problem-Solving Ability (HPSA)

Based on the research results, it is found that students in the high category have excellent problem-solving abilities. The subjects could answer all the questions correctly and precisely. This aligns with Pujiastuti's (2020) assertion that high-category students are good at designing strategies and solving problems confidently, carefully, and responsibly. In contrast, Zakiyah et al. (2021) suggest that high-category students can solve some problems correctly, but not all. However, in this study, high-category students demonstrated proficiency in almost all indicators of Polya's problem-solving stages.

In terms of problem-solving, HPSA subjects could choose the right strategy and write down the formula confidently to solve the problem. They worked on problems meticulously, sequentially, and systematically, relating them to everyday life. During the problem-solving stage, HPSA subjects could analyze information obtained earlier and solve problems well, obtaining accurate answers. In the final stage of reviewing the answers, they could provide conclusions from the preceding work process. This is consistent with Pujiastuti's (2020) research, which proves that high-category students' answers align with problem-solving indicators.

HPSA subjects successfully completed all four stages of Polya's problem-solving method: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) looking back. They could explain their answers confidently and correctly.

Moderate Category Mathematical Problem-Solving Ability (MPSA)

In the moderate category, MPSA subjects could understand the problems well, but they tended to be in a rush to solve them, lacking precision and neglecting to double-check. According to Akbar et al. (2018), students with moderate problem-solving abilities need special attention for development. Rini and Pramesti (2019) also suggest that student errors in solving problem-solving ability tests can be attributed to not being familiar with complex language, lack of accuracy leading to mistakes in using formulas, lack of diligence resulting in calculation errors, and an inability to manage time effectively.

Fuady (2020) emphasizes that problem-solving skills are closely related to students' ability to read and understand stories, present them in mathematical models, and perform calculations for non-routine problems. MPSA subjects did not explain the steps systematically but answered questions hurriedly. They answered only a few questions correctly, while the rest were answered hastily, focusing on providing an answer rather than ensuring its accuracy. In item 1, 2, and 3, MPSA subjects answered correctly, but errors occurred in each question as they did not check or draw conclusions from their work. According to Polya's theory, MPSA subjects had not fully understood the problem, planned the solution, or solved the problem properly. Similarly, they failed to meet the indicator of reviewing their answers.

Low Category Mathematical Problem-Solving Ability (LPSA)

Teachers need to pay close attention to low-category students as they made many errors in every process. Their understanding of mathematical concepts was low, making it challenging for them to solve problem-solving questions. LPSA subjects struggled to understand the problem from the beginning, indicating a need for guidance when faced with story problems. This aligns with Lestanti et al.'s (2016) statement that students are expected to understand the problem-solving process and have the skills to choose and find relevant conditions and concepts, find generalizations, plan, and organize.

Based on the problem-solving indicators, students' answers have not reached the maximum potential. Many students faced difficulties while solving the given problems. Purnomo and Prasetyo (2016) assert that problem-solving ability is the main goal of mathematics learning. Therefore, it should be introduced, practiced, and habituated to students as early as possible. Teachers must
provide problem-solving methods that are easy and interesting so that students can comprehend the given problems and come up with the best solutions for each of them.

Problem-solving ability is a fundamental competence that must be mastered by students (Rasyid et al., 2023). According to Polya's theory, students still need training and understanding of mathematical concepts so that they can analyze and model mathematical problems. This training will enable students to have a systematic understanding of the problem-solving process. Self-monitoring allows children to unite their findings during problem-solving operations and ensure the strategies they use (Ozsoy & Ataman, 2009).

In this study, many students skipped the planning stage of problem-solving. Both moderate and low-category students were too rushed in their work, even though this did not happen with every question. Another frequently skipped stage was reviewing the answers. This stage is crucial for helping students analyze mistakes made during the problem-solving process.

Rini & Pramesti (2019) mentioned several reasons for students' errors in solving problem-solving tests based on Polya's steps, such as being unfamiliar with complex language, lack of precision leading to mistakes in using formulas, lack of diligence resulting in calculation errors, and an inability to manage time effectively.

The novelty in this research lies in its unique object and data collection method compared to previous studies. Data was collected through documentation of students' problem-solving responses rather than presenting tasks to the students directly. The study included subjects from high, moderate, and low categories. The research provides valuable input for future studies, contributes to the development of knowledge in education, particularly in improving students' mathematical problem-solving abilities. The findings also serve as a benchmark for assessing students' confidence and abilities in solving mathematical problems and can be a reference for similar studies.

In conclusion, the subjects in the high, moderate, and low categories show differences in analyzing problem-solving questions. The ability to solve mathematical problems is crucial for learning and for practical applications in daily life. Therefore, mathematical problem-solving is essential in mathematics education, helping students face challenges in their current and future lives.

CONCLUSION

The conclusion of this research indicates that students with high criteria demonstrate a high level of understanding and possess strong motivation in answering problems. On the other hand, students with moderate criteria show an inability to fully comprehend the problems contained in the questions. These students struggle to capture the elements present in the questions, lack precision, and do not read the questions repeatedly, resulting in a limited understanding of the problems. Meanwhile, subjects in the low category exhibit a lack of precision, an inability to plan problem-solving effectively, and the capability to execute the plan fairly well but not accurately. Additionally, they are unable to review their answers and have not yet mastered the ability to complete the given problems correctly.

Based on the findings of this research, it is essential to implement a learning approach that focuses on strengthening conceptual understanding, problem-solving strategies, and comprehension of story problems. Furthermore, teachers should provide additional guidance, particularly for students with lower levels of ability, to ensure a better understanding of the concepts.

REFERENCES


