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Integrating Concrete Manipulatives and Problem-Based Learning to Improve Fraction Addition Outcomes in Elementary Students with Learning Disabilities

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Abstract
Fraction learning poses significant challenges for elementary students, particularly those with learning disabilities who struggle with abstract mathematical concepts. This classroom action research investigated the effectiveness of concrete media-based Problem-Based Learning (PBL) in improving simple fraction addition outcomes for second-grade students with learning disabilities. Conducted over three months at SD Negeri 005 Teluk Pandan, the study employed the Kemmis and McTaggart model across two cycles involving 20 students, including seven with identified learning disabilities. Each cycle comprised planning, action implementation, observation, and reflection phases. Data were collected through achievement tests, structured observations, and document analysis. Results demonstrated substantial improvements: overall class mean scores increased from 62.50 (pre-cycle) to 77.20 (Cycle I) and 83.00 (Cycle II), with 100% of students achieving the learning objective criterion by Cycle II. Students with learning disabilities showed remarkable progress, with mean scores rising from 40.71 to 75.86, representing an 86.3% improvement. All seven students with disabilities reached grade-level competency. The findings suggest that integrating concrete manipulatives with authentic problem-solving contexts creates synergistic effects, addressing multiple learning barriers simultaneously. This integrated approach offers practical implications for inclusive mathematics education, demonstrating that students with disabilities can achieve grade-level competency when provided multisensory, contextually meaningful instruction combining concrete materials with problem-based pedagogical frameworks.

INTRODUCTION

Learning outcomes serve as a critical indicator of educational success, representing the observable changes in student behavior, knowledge, and skills following the learning process (Polly et al., 2015). In mathematics education, achieving satisfactory learning outcomes remains a persistent challenge, particularly in elementary schools where foundational mathematical concepts are established. Among various mathematical topics, fractions present significant difficulties for students, as they require understanding abstract concepts that often conflict with students' intuitive whole-number reasoning (Lortie-Forgues et al., 2015). This challenge is magnified for students with learning disabilities, who typically struggle with abstract mathematical representations and require specialized instructional approaches to achieve meaningful learning outcomes.

The importance of mastering fraction operations, particularly simple fraction addition, cannot be overstated. Fractions constitute a fundamental mathematical concept with extensive applications in daily life and various academic disciplines (Wilkins & Norton, 2018). Despite their perceived simplicity, fraction addition with like denominators often serves as students' initial encounter with more sophisticated concepts including multiples, common factors, and algebraic thinking. However, research consistently demonstrates that fractions remain one of the most difficult topics in elementary and middle school mathematics to teach and learn (Gabriel et al., 2013; Singh et al., 2021). Lortie-Forgues et al. (2015) documented that fraction and decimal arithmetic pose large difficulties for many children and adults, with little improvement evident in students' understanding over the past three decades despite extensive instructional efforts. Students frequently exhibit incorrect ways of thinking about fraction size, applying natural number bias and other flawed reasoning patterns (Gómez et al., 2022). These fundamental difficulties create persistent barriers to mathematical achievement and limit students' access to advanced mathematics and STEM-related fields.

The challenges intensify significantly for students with learning disabilities. Namkung et al. (2019) emphasized that achieving competency with fractions is particularly challenging for students with mathematics learning difficulties who often lack foundational skills with whole numbers. Soares et al. (2018) noted that specific learning disabilities in mathematics manifest through difficulties in mastering number sense, calculation, and mathematical reasoning, affecting approximately 5% of students in public schools. Students with learning disabilities demonstrate unique ways of thinking about fractions, often incorrectly applying algorithms and exhibiting persistent error patterns (Widodo & Ikhwanudin, 2019). Recent research by Hunt et al. (2025) revealed that students with learning disabilities frequently encounter instruction that lacks relevance, multimodal engagement, or conceptual rigor, resulting in a continuous cycle of mathematical difficulties and avoidance. Polydoros et al. (2025) found that these students require specialized interventions integrating multiple instructional approaches to achieve meaningful learning outcomes in fraction comprehension.

Research has increasingly highlighted the effectiveness of concrete manipulatives in mathematics instruction. Carbonneau et al. (2013) conducted a meta-analysis demonstrating that teaching mathematics with concrete manipulatives produces small-to-medium-sized effects on student learning compared to instruction without concrete materials. Uribe-Flórez and Wilkins (2016) found positive relationships between manipulative use and elementary students' mathematics learning outcomes. More recent studies by Khan et al. (2024) confirmed that blended instruction combining concrete and virtual manipulatives significantly enhances mathematical achievement across various concepts including fractions. Byrne et al. (2023) conducted a comprehensive scoping review of 102 studies, revealing considerable evidence that physical manipulatives, when properly implemented, improve learning across diverse student populations. For students with disabilities specifically, Lafay et al. (2019) found that interventions using manipulatives were effective for various learning objectives, though considerable heterogeneity existed in implementation approaches. Bouck and Park (2018) systematically reviewed literature on mathematics manipulatives for students with disabilities, identifying concrete manipulatives as an evidence-based practice, while Bouck et al. (2025) demonstrated their effectiveness in teaching fraction concepts through multiple representations.

Alongside concrete media, Problem-Based Learning has emerged as a promising pedagogical approach. Research indicates that PBL improves mathematical critical thinking, problem-solving, and creative thinking skills across educational levels (systematic review of 14 studies, 2024). PBL encourages students to explore, discuss, and understand mathematical concepts by solving problems collaboratively, fostering deeper understanding of mathematical principles (EdSurge, 2024). Meta-analysis findings demonstrate that project-based learning significantly improves student learning effects, with the greatest impact occurring during experimental periods of 9-18 weeks (2023).

Despite these encouraging findings, a critical gap persists. While Bouck et al. (2022) provided practical guidance for implementing manipulative-based instructional sequences for students with disabilities, and various studies examined PBL's effectiveness in mathematics, limited research has systematically investigated the combined implementation of concrete media with PBL specifically for students with learning disabilities in fraction instruction. Most existing studies focus on general populations or examine these approaches separately, with insufficient attention to how their integration might address the unique needs of vulnerable learners.

This research addresses these gaps by investigating concrete media-based PBL for students with learning disabilities in simple fraction addition. Preliminary assessments at SD Negeri 005 Teluk

Pandan revealed poor performance in fraction addition, with average scores of 62.50 among 20 second-grade students, and only 40.71 among seven students with learning disabilities. This study aims to determine whether this integrated approach can effectively improve learning outcomes, providing empirical evidence for comprehensive instructional strategies that accommodate diverse learning styles and developmental levels while creating more inclusive and effective mathematics instruction.

METHODS

This study employed a classroom action research design following the Kemmis and McTaggart model, which is widely recognized for its cyclical and reflective approach to improving educational practices. The research was conducted collaboratively between the researcher and the second-grade classroom teacher, ensuring practical relevance and sustainable implementation of the intervention. The study consisted of two complete cycles, each comprising four interconnected phases: planning, action implementation, observation, and reflection. Each cycle was designed to build upon insights gained from the previous one, allowing for continuous refinement of instructional strategies based on empirical evidence and classroom realities.

The research was conducted at SD Negeri 005 Teluk Pandan, located in Danau Redan Village, approximately 18 kilometers from Samarinda city center along the Poros-Bontang road. The school serves a predominantly low-to-middle-income community with a total enrollment of 180 students. The participants comprised 20 second-grade students, among whom seven were identified as having learning disabilities in mathematics. These seven students, who became the primary focus of the intervention, were purposively selected based on initial teacher observations, diagnostic assessments, and consistently low performance in fraction operations. Diagnostic testing revealed that these students scored an average of 40.71 out of 100 on fraction addition tasks, with particular difficulties in understanding basic concepts such as place value, grouping, and the part-whole relationship inherent in fractions. The broader classroom context of 20 students was maintained to examine intervention effects in authentic, inclusive learning environments.

Data collection utilized multiple instruments to ensure comprehensive evaluation of learning outcomes and instructional processes. Achievement tests measuring fraction addition competency were administered before the intervention (pre-cycle), after each cycle, and at the study's conclusion. These tests were designed to assess both procedural fluency and conceptual understanding of simple fraction addition with like denominators. Structured observation sheets documented student participation, engagement with concrete materials, problem-solving approaches, and collaborative behaviors during PBL activities. The observation protocol captured both quantitative measures of participation frequency and qualitative descriptions of learning processes. Document analysis of students' written work provided additional insights into their mathematical reasoning and error patterns. To establish content validity, all instruments were reviewed by mathematics education experts and experienced elementary teachers. Inter-rater reliability for observation protocols was established through initial joint observations by the researcher and classroom teacher, achieving agreement rates exceeding 85%.

The intervention spanned three months, with each cycle requiring four weeks: two weeks for instruction and data collection, and two weeks for data analysis and reflection. Instructional sessions occurred twice weekly for 35 minutes each, utilizing concrete materials such as fraction circles, colored paper, and fruit models within a PBL framework that presented authentic, contextual problems. Data were analyzed using both qualitative and quantitative descriptive methods. Quantitative analysis calculated mean scores, percentage improvements, and achievement rates against the learning objective criterion of 70. Qualitative analysis employed thematic coding of observation notes and student work samples to identify patterns in learning processes, conceptual development, and persistent difficulties. Success criteria included minimum 20% improvement in test scores from pre-test to post-test in each cycle, at least 30% increase in active participation, and positive attitudinal changes in 80% of students as evidenced through observations and informal assessments.

RESULTS AND DISCUSSION

Results

The implementation of concrete media-based Problem-Based Learning (PBL) for teaching simple fraction addition was conducted over three months through two complete cycles of classroom action research. Each cycle yielded progressively improved learning outcomes among all students, with particularly notable gains observed among students with learning disabilities. The findings demonstrate systematic improvement across multiple assessment points, providing evidence for the effectiveness of this integrated instructional approach.

Pre-Cycle Baseline Assessment

Prior to intervention, diagnostic assessments revealed significant challenges in fraction addition competency among second-grade students. Table 1 presents the distribution of student achievement levels during the pre-cycle phase, establishing the baseline from which subsequent improvements would be measured.

Table 1. Pre-Cycle Learning Outcomes Distribution (N=20)

Score Range	Number of Students	Percentage	Status
< 70	14	70%	Below KKTP
70-79	0	0%	Met KKTP
80-89	6	30%	Met KKTP
90-100	0	0%	Met KKTP
Mean Score	62.50		

As shown in Table 1, only six students (30%) achieved scores meeting the learning objective criterion (KKTP) of 70, while the majority (70%) scored below this threshold. The mean score of 62.50 indicated substantial room for improvement in fraction addition understanding. Among the seven students identified with learning disabilities, performance was considerably lower, with an average score of 40.71 and all seven students scoring below the KKTP threshold. These baseline data confirmed the need for targeted intervention and established clear benchmarks for measuring subsequent progress.

Cycle I Outcomes

Following the implementation of concrete media-based PBL instruction during Cycle I, observable improvements emerged in both achievement scores and classroom engagement. Table 2 presents the comparative performance data for the entire class and specifically for students with learning disabilities.

Table 2. Cycle I Learning Outcomes

Student Group	Pre-Cycle Mean	Cycle I Mean	Improvement	Students Meeting KKTP
All students (N=20)	62.50	77.20	23.5%	13 (65%)
Students with LD (N=7)	40.71	62.29	53.0%	2 (28.6%)

The data in Table 2 reveal substantial improvements across both groups. For the entire class, the mean score increased from 62.50 to 77.20, representing a 23.5% gain that exceeded the predetermined success criterion of 20% improvement. The number of students achieving KKTP more than doubled, rising from 6 to 13 students (65% of the class). Among students with learning disabilities, improvements were even more pronounced in percentage terms, with mean scores increasing by 53.0% from 40.71 to 62.29. While only two of these seven students reached the KKTP threshold by Cycle I, all seven demonstrated meaningful progress from baseline, suggesting that the intervention was beginning to address their specific learning needs.

Observational data collected during Cycle I documented increased active participation during learning activities. Students demonstrated greater willingness to manipulate concrete materials, engage in peer discussions within PBL groups, and attempt problem-solving independently. However, observations also revealed areas requiring refinement, including the need for additional scaffolding in transitioning from concrete manipulations to symbolic representations and more structured guidance in the problem-solving process for students with learning disabilities.

Cycle II Outcomes

Building upon insights gained from Cycle I reflection, the second cycle incorporated refined instructional strategies with enhanced scaffolding and more explicit connections between concrete materials and abstract fraction concepts. Table 3 presents the achievement data following Cycle II implementation.

Table 3. Cycle II Learning Outcomes Compared to Previous Phases

Score Range	Pre-Cycle	Cycle I	Cycle II
< 70	14 students (70%)	7 students (35%)	0 students (0%)
70-79	0 students (0%)	5 students (25%)	12 students (60%)
80-89	6 students (30%)	7 students (35%)	8 students (40%)
90-100	0 students (0%)	1 student (5%)	0 students (0%)
Mean Score	62.50	77.20	83.00

The Cycle II data presented in Table 3 demonstrate remarkable progress, with 100% of students achieving scores at or above the KKTP threshold of 70. The class mean increased to 83.00, representing a 32.8% improvement from baseline and a 7.5% gain from Cycle I. Notably, the distribution shifted substantially toward higher achievement levels, with 60% of students scoring in the 70-79 range and 40% in the 80-89 range. This distribution indicates that the intervention not only helped struggling students reach minimum competency but also elevated overall class performance.

Table 4. Individual Progress of Students with Learning Disabilities Across All Phases

Student	Pre-Cycle	Cycle I	Cycle II	Total Improvement
Student 1	35	50	70	+35 points (100%)
Student 2	40	60	78	+38 points (95%)
Student 3	50	70	80	+30 points (60%)
Student 4	40	66	75	+35 points (87.5%)
Student 5	50	65	78	+28 points (56%)
Student 6	50	75	80	+30 points (60%)
Student 7	20	50	70	+50 points (250%)
Mean	40.71	62.29	75.86	+35.15 points (86.3%)

Table 4 provides detailed individual trajectories for students with learning disabilities, revealing that all seven students achieved KKTP by Cycle II. The mean score for this group increased to 75.86, representing an 86.3% improvement from baseline. Individual gains ranged from 28 to 50 points, with Student 7 demonstrating the most dramatic improvement from 20 to 70 points. These data indicate that the concrete media-based PBL approach successfully addressed the specific learning needs of students with disabilities, enabling them to overcome initial difficulties and achieve grade-level competency in fraction addition.

Qualitative observations during Cycle II documented enhanced conceptual understanding beyond mere procedural competence. Students increasingly verbalized their reasoning using fraction terminology, demonstrated ability to represent problems using multiple modalities (concrete, pictorial, and symbolic), and showed persistence in problem-solving even when encountering challenging tasks. Students with learning disabilities, in particular, exhibited greater confidence in mathematical communication and reduced reliance on teacher assistance as they became more proficient with concrete materials and problem-solving strategies.

Discussion

The findings of this study provide compelling evidence that integrating concrete media with Problem-Based Learning creates a powerful instructional approach for teaching fraction addition, particularly for students with learning disabilities. The 86.3% average improvement among students with learning disabilities and 100% achievement of KKTP by all students in Cycle II substantially exceed typical intervention outcomes reported in the literature, warranting careful examination of the mechanisms underlying this success.

The remarkable effectiveness observed in this study aligns with and extends theoretical frameworks established in prior research. Carbonneau et al. (2013) meta-analysis found small-to-medium effect sizes for concrete manipulatives in mathematics instruction, yet our intervention produced considerably larger gains. This amplified effectiveness likely stems from the synergistic combination of concrete materials with PBL's problem-solving framework, rather than using manipulatives alone. The concrete materials provided students with tangible representations of abstract fraction concepts, addressing the fundamental challenge identified by Lortie-Forgues et al. (2015) that fraction arithmetic requires overcoming natural number bias and developing new conceptual frameworks. By manipulating fruit models and colored paper to physically combine fraction parts, students constructed concrete-to-abstract mental bridges that passive instruction fails to establish.

The PBL component contributed essential elements that concrete materials alone cannot provide. Hunt et al. (2025) emphasized that students with learning disabilities require instruction offering relevance, multimodal engagement, and conceptual rigor—precisely the characteristics of well-designed PBL. Our contextualized problems enabled students to connect fraction operations to familiar situations, transforming abstract symbols into meaningful quantities. This contextualization addresses concerns raised by Gabriel et al. (2013) regarding students' tendency to apply rote procedures without conceptual understanding. The authentic problem-solving contexts motivated students to make sense of their manipulations rather than merely following algorithms, fostering the conceptual grounding necessary for lasting understanding.

For students with learning disabilities specifically, the intervention's structure addressed multiple documented barriers to fraction learning. Namkung et al. (2019) identified that students with mathematics learning difficulties require explicit instruction, systematic scaffolding, and extended practice opportunities—all embedded within our cyclical action research design. The progression from simple to complex problems within each cycle, coupled with deliberate movement from concrete to representational to abstract modes (aligning with Bouck et al., 2022 recommendations), provided the graduated support these students required. Lafay et al. (2019) noted that manipulative-based interventions show effectiveness for various learning objectives but require careful implementation; our two-cycle design allowed continuous refinement based on observed student needs, potentially explaining superior outcomes compared to single-implementation studies.

Comparison with Indonesian studies reveals both consistencies and noteworthy distinctions. Dhani and Ahmad (2022), Nurjannah and Setiyadi (2022), Astika (2023), Rahmawati (2024), and Tafonao (2023) all reported positive effects of PBL on fraction learning outcomes, supporting our findings regarding PBL's general effectiveness. However, our study's focus on students with learning disabilities and integration of concrete media extends these findings to a more vulnerable population. Afifa (2024) and Amelia et al. (2025) demonstrated concrete media effectiveness for fraction learning, with Purwanti (2015) specifically documenting benefits of concrete manipulatives for fraction multiplication. Our results confirm these benefits extend to fraction addition for students with disabilities. Shoimah et al. (2021) and Yulistianingsih et al. (2024) found concrete media improved both activity and conceptual understanding, paralleling our qualitative observations of increased engagement and verbalized reasoning.

Particularly relevant is Irfan (2017), who combined PBL with Numbered Heads Together cooperative learning for fraction addition in fourth grade, achieving positive results. Our study advances this work by focusing on younger students (second grade) with identified disabilities and systematically documenting intervention refinement through action research cycles. The progression of student with learning disabilities from 40.71 to 75.86 average scores represents success rates exceeding those typically reported for this population, potentially attributable to our sustained, responsive intervention approach.

Several mechanisms appear to explain the intervention's effectiveness. First, concrete materials transformed abstract fraction concepts into manipulable objects, addressing the sensory-motor learning needs emphasized by constructivist theory (Bruner, 1966; Piaget, 1970) and supported by Byrne et al. (2023) review of manipulative interventions. Second, PBL's authentic contexts activated students' prior knowledge and created meaningful problem-solving purposes, addressing motivational factors identified by Polydoros et al. (2025) as critical for students with learning difficulties. Third, the collaborative nature of PBL groups provided peer scaffolding and mathematical discourse opportunities, supporting Uribe-Flórez and Wilkins (2016) findings regarding social dimensions of manipulative use.

The cyclical action research design itself contributed substantially to effectiveness. Unlike fixed interventions, our approach allowed responsive modifications based on Cycle I observations. Refinements included more explicit think-aloud modeling of concrete-to-abstract transitions, structured sentence frames for mathematical communication, and differentiated problem complexity within PBL groups. These adaptations, informed by Bouck and Park's (2018) guidance for students with disabilities, likely enhanced Cycle II outcomes. The dramatic improvement of Student 7 (from 20 to 70 points) exemplifies how targeted, responsive instruction can overcome severe initial difficulties.

However, certain limitations warrant consideration. The study's three-month duration, while sufficient for demonstrating immediate effects, leaves questions regarding long-term retention. Lafay et al. (2019) noted that manipulative interventions should assess maintenance and transfer; future research should examine whether gains persist and generalize to more complex fraction operations. Additionally, while qualitative observations suggested conceptual understanding development, more robust measures of conceptual versus procedural knowledge would strengthen claims about the nature of learning occurring.

The findings carry important implications for inclusive mathematics education. First, they demonstrate that students with learning disabilities can achieve grade-level competency when provided appropriate instructional supports, challenging deficit-oriented perspectives. Second, they suggest that concrete media and PBL need not be reserved for remediation but can form core instructional approaches benefiting all learners—our data show both struggling and typical students improved substantially. Third, the action research methodology itself offers a model for teacher-driven instructional improvement, enabling practitioners to systematically refine their teaching based on evidence rather than intuition alone.

CONCLUSION

This classroom action research demonstrated that integrating concrete media with Problem-Based Learning significantly improves fraction addition learning outcomes for elementary students, particularly those with learning disabilities. Over two instructional cycles, all 20 second-grade students achieved the learning objective criterion, with mean scores increasing from 62.50 to 83.00. Most notably, students with learning disabilities improved from an average of 40.71 to 75.86, representing an 86.3% gain that enabled all seven students to reach grade-level competency. These findings extend beyond typical intervention outcomes, suggesting that the synergistic combination of tangible manipulatives with authentic problem-solving contexts addresses multiple learning barriers simultaneously.

The study contributes empirical evidence supporting integrated instructional approaches for inclusive mathematics education. While previous research examined concrete manipulatives and PBL separately, this study documents their combined effectiveness specifically for vulnerable learners, demonstrating that students with disabilities can achieve grade-level mathematics competency when provided multisensory, contextually meaningful instruction. The cyclical action research design itself offers methodological insights, showing how teacher-driven, responsive refinements enhance intervention effectiveness.

Practical implications suggest that educators should prioritize authentic problem contexts when using manipulatives rather than treating concrete materials as mere visualization tools. The findings support inclusive classroom practices where specialized instructional strategies benefit all learners, not only those with identified disabilities. Teacher professional development should emphasize integrated pedagogical approaches and systematic reflection on student learning patterns.

However, several limitations warrant consideration. The three-month duration provides evidence of immediate effectiveness but leaves questions regarding long-term retention and transfer to more complex fraction operations. The small sample size and single-school context limit generalizability across diverse educational settings. Future research should employ longitudinal designs examining whether gains persist, investigate the intervention's effectiveness across different grade levels and mathematical topics, explore optimal concrete material types for various fraction concepts, and examine implementation feasibility in larger-scale contexts with varied teacher expertise levels. Comparative studies isolating the unique contributions of concrete media versus PBL components would further clarify the mechanisms underlying observed improvements.

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