

Using Cognitive Diagnostic Assessment to Inform Differentiated Instruction in Elementary Place Value Concepts

Dhesta Nurdana Puspita*

Faculty of Education, State University of Surabaya, Surabaya, Indonesia

Wiryanto

Faculty of Education, State University of Surabaya, Surabaya, Indonesia

Tatag Yuli Eko Siswono

Faculty of Mathematics and Natural Sciences, State University of Surabaya, Surabaya, Indonesia

*Corresponding Author: 24010855153@mhs.unesa.ac.id

Abstract

Place value understanding is a fundamental prerequisite for advanced mathematical learning, yet elementary students often demonstrate heterogeneous comprehension that challenges uniform teaching approaches. This study examines the use of cognitive diagnostic assessment (CDA) to identify student learning profiles and inform differentiated instruction in Indonesian elementary mathematics. A mixed-methods descriptive design involved 12 fourth-grade students at SD Negeri Jubellor, East Java. Data were collected through structured interviews and a five-item diagnostic test on place value concepts. Psychometric properties were analyzed using ANATES, while student performance was categorized into high ($\geq 98\%$), moderate (64–97%), and low ($< 64\%$) understanding levels. The assessment showed strong psychometric quality with item correlations between 0.815–0.876 ($p < 0.01$) and high reliability ($\alpha = 0.87$). Difficulty analysis indicated balanced distribution, with one very easy item (20%) and four moderately difficult items (80%). Results revealed heterogeneous profiles: 25% low understanding, 42% moderate, and 33% high. Students with low understanding struggled with reading multi-digit numbers and place value beyond thousands. Those at the moderate level demonstrated competency up to ten thousands but faced difficulties at hundred thousands and contextual applications. High-achieving students mastered place value comprehensively, including real-world applications. These findings demonstrate that CDA not only identifies specific learning gaps but also provides actionable insights for planning targeted instructional pathways. Strengthening this diagnostic-instructional alignment is essential to ensure responsive teaching that supports all learner profiles, particularly in bridging conceptual gaps among students at the moderate and low understanding levels.

Keywords

Cognitive Diagnostic Assessment
Place Value
Differentiated Learning
Elementary Mathematics
Learning Needs Assessment

Article History

Received 2025-09-21
Accepted 2025-12-20

Copyright © 2026 by Author(s).
This is an open access article
under the [CC BY-SA](https://creativecommons.org/licenses/by-sa/4.0/) license.

INTRODUCTION

Mathematics education in elementary settings serves as the foundational pillar for students' long-term academic success and scientific literacy development. Among the core mathematical concepts, place value understanding represents a critical gateway skill that influences students' ability to comprehend advanced numerical operations, algebraic thinking, and mathematical reasoning (Basargekar & Lillard, 2021). The significance of place value extends beyond computational proficiency to encompass conceptual understanding of the base-ten number system, multiplicative relationships between digit positions, and the abstract notion of positional notation (Jannah et al., 2024). Research consistently demonstrates that students with robust place value comprehension demonstrate superior performance in multi-digit arithmetic, fraction concepts, and decimal operations (Benden & Lauermann, 2022). Contemporary educational frameworks emphasize the importance of diagnostic assessment to identify student readiness levels and inform evidence-based instructional decisions (Isnah et al., 2024). The integration of assessment and instruction represents a paradigm shift toward responsive teaching that adapts to learner diversity rather than assuming homogeneous student populations (Romlah et al., 2025). International comparative studies reveal significant

variations in place value instruction effectiveness across educational systems, highlighting the need for culturally responsive diagnostic practices (Ramdan & Husni, 2024). The theoretical foundation for place value instruction draws from constructivist learning theory, emphasizing the importance of building upon existing student knowledge structures and addressing misconceptions systematically (Piaget, 1970).

The Indonesian educational context presents unique challenges for mathematics instruction, including diverse socioeconomic backgrounds, varying levels of parental educational support, and limited access to supplementary learning resources. Elementary schools in Indonesia serve highly heterogeneous student populations with significant disparities in prior mathematical knowledge and home language experiences (Sujinah et al., 2024). The national curriculum framework emphasizes place value instruction across multiple grade levels, progressing from single-digit recognition in early grades to hundred thousands comprehension by fifth grade (Wulansari et al., 2023). However, implementation challenges include large class sizes, limited instructional materials, and insufficient teacher preparation in diagnostic assessment practices. Recent educational reforms promote student-centered learning approaches and differentiated instruction, requiring teachers to develop new competencies in assessment and responsive teaching (Fauzan, 2025). The cultural context of Indonesian education traditionally emphasizes collective learning and teacher authority, creating tensions with individualized instructional approaches. Research indicates that Indonesian elementary students demonstrate particular challenges with place value concepts involving zero as a placeholder and multi-digit number comparison tasks (Hasibuan et al., 2024). The integration of technology-enhanced assessment tools remains limited in many Indonesian schools, necessitating practical diagnostic approaches that function within existing resource constraints.

Despite the recognized importance of place value concepts, substantial evidence indicates that many elementary students worldwide struggle with fundamental aspects of positional notation understanding. Common misconceptions include treating multi-digit numbers as collections of single digits rather than understanding place value relationships, confusion about zero's role as a placeholder, and difficulties with number comparison tasks involving different digit quantities (Wijayanti, 2023). Research documents persistent patterns of place value errors across grade levels, suggesting that traditional instructional approaches may inadequately address conceptual development (Thapliyal et al., 2022). Students often demonstrate procedural competency in basic operations while lacking conceptual understanding of underlying place value principles. The disconnect between procedural and conceptual knowledge creates fragile learning that breaks down when students encounter novel problems or advanced mathematical concepts (Star, 2005). International assessment data reveals that place value difficulties are particularly pronounced among students from disadvantaged socioeconomic backgrounds, highlighting equity concerns in mathematics education (Paulsen & Valdivia, 2022). Cross-cultural research indicates that place value instruction effectiveness varies significantly across different number naming systems and cultural contexts (Miller, 1956). The complexity of place value concepts requires carefully sequenced instruction that addresses prerequisite skills systematically and provides multiple representational approaches.

Preliminary investigations conducted by the research team at various Indonesian elementary schools revealed concerning patterns in student place value understanding (Anggoro et al., 2024). Formative assessment data from fifth-grade students indicated that only 40% achieved minimum competency standards on whole number tasks, with 60% scoring below acceptable levels. Diagnostic interviews revealed that while most students could identify and write numbers through four digits, significant difficulties emerged with five-digit numbers and beyond. Error analysis identified three primary categories: number naming mistakes, symbolic representation errors, and place value determination inaccuracies. These findings align with international research while highlighting specific challenges within the Indonesian educational context. The heterogeneous nature of student understanding levels, ranging from basic digit recognition to sophisticated place value manipulation, underscores the necessity for differentiated instructional approaches. Traditional whole-class

instruction assumes uniform student readiness levels, failing to address the diverse learning needs evident in typical elementary classrooms (Drijvers et al., 2021). The complexity of diagnosing student understanding requires systematic assessment approaches that move beyond simple correct/incorrect scoring to examine underlying conceptual frameworks.

Differentiated instruction emerges as a promising pedagogical approach to address the heterogeneous learning needs characteristic of elementary mathematics classrooms. This instructional framework involves modifying content, process, product, or learning environment based on student readiness levels, interests, and learning profiles (Tomlinson & Jackson, 2021). Theoretical foundations for differentiated instruction draw from multiple intelligences theory, learning style research, and zone of proximal development concepts (Gardner, 2011). Effective differentiated instruction requires comprehensive understanding of individual student capabilities, learning preferences, and motivational factors. Research demonstrates that differentiated approaches can significantly improve student achievement outcomes while reducing achievement gaps between high and low performers (Benden & Lauermann, 2022). The implementation of differentiated instruction presents challenges including time constraints, classroom management complexity, and the need for diverse instructional materials. Contemporary educational technology offers potential solutions through adaptive learning platforms and data analytics tools that support individualized instruction. However, successful differentiated instruction ultimately depends on teacher expertise in assessment, curriculum design, and flexible grouping strategies. The cultural context of differentiated instruction implementation requires consideration of local educational values, family expectations, and institutional constraints.

Cognitive diagnostic assessment represents a sophisticated measurement approach designed to provide detailed information about student knowledge states and skill mastery levels (Isnah et al., 2024). Unlike traditional assessments that focus primarily on overall achievement scores, cognitive diagnostic models analyze student response patterns to identify specific strengths and weaknesses within content domains (Plevris et al., 2023). The theoretical foundation for cognitive diagnostic assessment draws from cognitive psychology, educational measurement theory, and artificial intelligence research. Fine-grained diagnostic information enables teachers to design targeted interventions that address specific learning needs rather than implementing broad remediation strategies. Research demonstrates that cognitive diagnostic feedback significantly improves both student learning outcomes and teacher instructional effectiveness (Basargekar & Lillard, 2021). The development of cognitive diagnostic assessments requires careful attention to construct definition, item design, and validation procedures. Contemporary advances in psychometric modeling enable more sophisticated diagnostic capabilities while maintaining practical feasibility for classroom implementation. The integration of cognitive diagnostic assessment with instructional design represents an emerging area of research with significant potential for improving educational effectiveness.

This research addresses a critical gap in understanding how cognitive diagnostic assessment can inform differentiated learning design in elementary mathematics education, particularly within developing educational contexts. While extensive research exists on both diagnostic assessment and differentiated instruction independently, limited studies examine their systematic integration for place value instruction. The investigation contributes to the growing literature on evidence-based mathematics instruction by demonstrating practical approaches for implementing diagnostic assessment within resource-constrained environments. Research findings provide empirical foundation for policy discussions regarding assessment integration and teacher professional development in elementary mathematics education. The study's focus on Indonesian educational contexts addresses the need for culturally responsive research that considers local constraints and opportunities. Methodological contributions include the development of practical diagnostic assessment protocols that can be adapted to similar educational settings throughout developing regions. The investigation's emphasis on place value concepts addresses fundamental mathematical understanding that influences

long-term academic success trajectories. Results inform teacher education programs regarding the integration of assessment competencies with instructional design skills.

METHODS

This investigation employed a mixed-methods descriptive research design combining quantitative psychometric analysis with qualitative diagnostic assessment to examine student place value understanding comprehensively. The philosophical foundation draws from pragmatist epistemology, emphasizing the practical utility of research findings for educational improvement (Creswell & Plano Clark, 2023). Mixed-methods approaches enable triangulation of quantitative performance data with qualitative insights into student thinking processes, providing richer understanding of learning phenomena than either method alone (Tashakkori & Teddlie, 2008). The descriptive nature of the study focuses on characterizing existing conditions rather than implementing experimental interventions, appropriate for establishing baseline understanding of student capabilities. Quantitative components examine psychometric properties of diagnostic instruments and analyze performance patterns across student populations. Qualitative elements explore individual student reasoning processes and contextual factors influencing learning outcomes. The sequential explanatory design prioritizes quantitative data collection and analysis, followed by qualitative investigation to explain and elaborate quantitative findings.

The study was conducted at SD Negeri Jubellor, located in Sugio District, Lamongan Regency, East Java Province, Indonesia, during the 2024 academic year. The school serves a rural community with mixed socioeconomic backgrounds, representing typical Indonesian elementary educational settings. Participant selection employed purposive sampling based on accessibility, grade-level appropriateness, and parental consent availability. The sample comprised 12 fourth-grade students (ages 9-10 years) including 7 female and 5 male participants, reflecting natural classroom composition. Inclusion criteria required regular school attendance, Indonesian language proficiency, and absence of diagnosed learning disabilities that might confound place value assessment. Exclusion criteria eliminated students with frequent absences or significant language barriers that could affect assessment validity. The sample size aligns with descriptive research requirements while acknowledging limitations for statistical generalization. Demographic diversity within the sample includes students from agricultural families (58%), small business families (25%), and government employee families (17%), representing typical rural Indonesian community composition.

Ethical protocols followed Indonesian educational research guidelines and international standards for research involving minors. Institutional approval was obtained from the school administration and local education office prior to data collection. Informed consent procedures included written parental permission and student assent for all participants. Confidentiality protections ensured participant anonymity through numerical coding systems and secure data storage protocols. Student welfare considerations prioritized psychological comfort during assessment administration, with provisions for discontinuation if distress occurred. Data collection procedures minimized disruption to regular instructional time and academic progress. Results reporting maintains participant anonymity while providing sufficient detail for research validity. All assessment materials and procedures underwent review for cultural sensitivity and age-appropriateness.

Instrumentation

Cognitive Diagnostic Assessment

The primary instrument consisted of a five-item cognitive diagnostic test specifically designed to evaluate place value understanding across multiple competency levels. Item development followed established cognitive diagnostic assessment principles including construct clarity, cognitive model alignment, and multiple solution pathway accommodation. The assessment employed essay format questions to enable analysis of student reasoning processes and identification of specific misconceptions. Item content progressed systematically from basic place value identification to

complex applications involving multi-digit number manipulation and real-world contexts. Each item was designed to assess specific cognitive attributes related to place value understanding:

- Item 1: Basic place value identification in four-digit numbers
- Item 2: Multi-digit number comparison and ordering
- Item 3: Place value determination in five-digit numbers
- Item 4: Contextual application of place value concepts
- Item 5: Understanding of zero's role in positional notation

Scoring rubrics employed analytical frameworks that assigned partial credit for partially correct responses and identified common error patterns. Inter-rater reliability procedures involved independent scoring by two trained evaluators with disagreements resolved through discussion and consensus.

Interview Protocol

Semi-structured interview protocols provided supplementary data regarding student thinking processes, confidence levels, and metacognitive awareness during problem-solving activities. Interview questions employed think-aloud procedures and follow-up probes to elicit detailed explanations of student reasoning. The protocol included standardized questions while allowing flexibility for exploring unexpected student responses or interesting thinking patterns. Interview duration averaged 15-20 minutes per participant to maintain engagement while gathering comprehensive data.

Data Collection Procedures

Data collection followed a systematic three-phase protocol designed to maximize data quality while minimizing student stress and instructional disruption. Phase 1 involved preliminary interviews with the classroom teacher (Mrs. SM) to gather contextual information about student backgrounds, prior mathematical experiences, and classroom instructional approaches. Phase 2 implemented the cognitive diagnostic assessment under standardized conditions including consistent instructions, adequate time allocation, and comfortable testing environment. Phase 3 conducted individual student interviews to explore reasoning processes and gather qualitative insights into assessment responses.

Assessment administration occurred during regular mathematics instruction periods to maintain ecological validity and student comfort. Standardized instructions emphasized that the assessment was for research purposes rather than grading, reducing performance anxiety. Students received adequate time to complete all items without time pressure that might compromise response quality. Individual interviews were conducted in a private setting to encourage honest responses and detailed explanations.

Data Analysis Framework

Psychometric Analysis

Psychometric evaluation employed ANATES (Analysis Test) software for comprehensive examination of validity, reliability, and difficulty characteristics (Wiguna, 2024). Validity analysis utilized point-biserial correlation coefficients between individual item scores and total test scores, with significance testing at $\alpha = 0.05$ level. Items achieving correlation coefficients ≥ 0.576 were considered valid based on critical values for the sample size. Reliability analysis employed Cronbach's alpha coefficient to assess internal consistency, with values ≥ 0.70 indicating acceptable reliability (Nunnally & Bernstein, 1994). Difficulty analysis calculated the proportion of students answering each item correctly, with interpretation categories: easy ($P > 0.70$), moderate ($0.30 \leq P \leq 0.70$), and difficult ($P < 0.30$).

Performance Analysis

Student performance analysis utilized the formula: Student Score = (Total Points Earned / Maximum Possible Points) $\times 100$. Performance categories followed established diagnostic assessment frameworks: High Understanding ($\geq 98\%$ - Complete Mastery), Moderate Understanding (64-97% - Partial Mastery), and Low Understanding ($< 64\%$ - Inadequate Mastery). These categories align with

competency-based assessment principles and provide actionable information for instructional planning.

Qualitative Analysis

Interview data underwent thematic analysis following Braun & Clarke (2019) six-phase framework: familiarization, initial coding, theme searching, theme reviewing, theme defining, and report writing. Coding procedures emphasized student reasoning processes, misconception patterns, and metacognitive awareness indicators. Theme development focused on understanding factors that contribute to place value difficulties and successful learning strategies.

Table 1. Data Analysis Framework Summary

Analysis Component	Method	Software/Tool	Interpretation Framework
Item Validity	Point-biserial correlation	ANATES	$r \geq 0.576 = \text{Valid}$
Reliability	Cronbach's Alpha	ANATES	$\alpha \geq 0.70 = \text{Acceptable}$
Item Difficulty	Proportion correct	ANATES	Easy/Moderate/Difficult
Student Performance	Percentage scores	Excel	High/Moderate/Low
Interview Data	Thematic analysis	Manual coding	Reasoning patterns

RESULTS AND DISCUSSION

Results

Psychometric Properties of the Assessment Instrument

The cognitive diagnostic assessment demonstrated robust psychometric properties across all evaluated dimensions, confirming its appropriateness for measuring place value understanding in the target population. Validity analysis revealed strong statistical relationships between individual items and total test scores, with all correlation coefficients exceeding the critical threshold for significance ($r = 0.576$, $p < 0.05$). Item validity coefficients ranged from 0.815 to 0.876, indicating very strong relationships between individual item performance and overall test achievement. These findings suggest that all items contribute meaningfully to the overall construct of place value understanding and demonstrate appropriate discrimination among students with different ability levels.

Reliability analysis yielded a Cronbach's alpha coefficient of 0.87, substantially exceeding the minimum threshold of 0.70 for acceptable internal consistency (Nunnally & Bernstein, 1994). This high reliability indicates that the assessment items measure the same underlying construct consistently and that test scores would likely remain stable across similar testing conditions. The reliability coefficient falls within the 0.80-0.90 range considered optimal for diagnostic assessments, providing confidence in the consistency of measurement outcomes.

Table 2. Psychometric Analysis Results

Item	Validity		Difficulty	
	Correlation	Significance	Percentage Correct	Category
1	0.815	Very Significant	93.33%	Very Easy
2	0.876	Very Significant	66.67%	Moderate
3	0.866	Very Significant	61.11%	Moderate
4	0.815	Very Significant	66.67%	Moderate
5	0.867	Very Significant	50.00%	Moderate

Test Reliability (Cronbach's α) = 0.87 * $p < 0.01$

Item difficulty analysis revealed strategic distribution across difficulty levels, with one very easy item (20%) and four moderate difficulty items (80%). Item 1, focusing on basic place value identification, achieved the highest success rate (93.33%), indicating that most students possess fundamental place value recognition skills. Items 2-4 demonstrated similar moderate difficulty levels (61-67% success rates), suggesting appropriate challenge levels for diagnostic discrimination. Item 5 recorded the lowest success rate (50.00%), reflecting the complexity of understanding zero's function in positional notation systems.

The difficulty distribution aligns with optimal diagnostic assessment design principles, providing sufficient range to identify students across different ability levels while avoiding extremely difficult

items that might discourage student effort (Hambleton et al., 1991). The predominance of moderate difficulty items ensures meaningful discrimination among students while maintaining accessibility for the majority of participants.

Student Performance Analysis

Individual item performance analysis revealed distinct patterns of success and difficulty across the five assessment items. Figure 1 illustrates the percentage of students achieving correct responses for each item, highlighting areas of relative strength and challenge within place value understanding.



Figure 1. Student Performance Distribution Across Individual Assessment Items

The performance pattern indicates that while most students demonstrate basic place value recognition skills (Item 1), substantial challenges emerge with more complex applications. Items 2-4 show relatively consistent performance levels, suggesting that approximately one-third of students struggle with intermediate place value concepts. Item 5's lower performance reflects the conceptual complexity of understanding zero's dual role as both a number and a positional placeholder.

Error analysis revealed common misconception patterns across items:

- Item 2: Students frequently confused digit quantity with number magnitude
- Item 3: Five-digit numbers presented particular challenges for place value identification
- Item 4: Contextual applications required additional cognitive processing beyond basic number recognition
- Item 5: Zero's function proved conceptually difficult, with many students viewing it simply as "nothing"

Overall student performance distribution demonstrates the heterogeneous nature of place value understanding within the classroom population. Performance categories were established based on total assessment scores using established diagnostic frameworks:

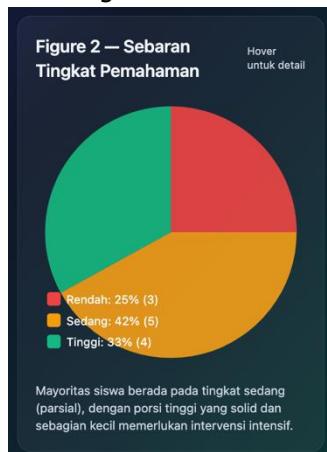


Figure 2. Distribution of Student Understanding Levels in Place Value Assessment

The distribution reveals that 42% of students demonstrate moderate understanding (partial mastery), representing the largest performance category. These students typically succeeded with basic place value identification and some multi-digit number tasks but struggled with complex applications and zero's function. 33% of students achieved high understanding (complete mastery), demonstrating comprehensive place value knowledge across all assessed competencies. 25% of students showed low understanding (inadequate mastery), indicating fundamental gaps in place value concepts requiring intensive intervention.

Qualitative Insights from Student Interviews

Interview data provided rich insights into student thinking processes and revealed underlying factors contributing to place value difficulties. Thematic analysis identified five major themes characterizing student understanding patterns:

- 1) Students frequently demonstrated procedural competency in place value identification without corresponding conceptual understanding of underlying principles. Many students could correctly identify place values through memorized patterns but struggled to explain why positional notation functions as it does. This finding aligns with research indicating that procedural and conceptual knowledge develop along different trajectories.
- 2) A significant subset of students confused the number of digits with numerical magnitude, believing that longer numbers are automatically larger regardless of digit values. For example, some students incorrectly identified 1,000 as smaller than 999 because "it has a zero in it."
- 3) The role of zero as a placeholder proved particularly challenging, with students expressing various misconceptions including "zero means nothing," "zero doesn't count," and confusion about when zeros can be ignored in number reading versus place value determination.
- 4) While many students succeeded with abstract number tasks, they struggled to apply place value knowledge in real-world contexts. This suggests that transfer of learning requires explicit instructional attention rather than automatic generalization.
- 5) Student confidence levels varied significantly and correlated with performance outcomes. Students expressing higher confidence demonstrated greater willingness to attempt challenging items and provide detailed explanations of their reasoning.

Discussion

The robust psychometric properties demonstrated by the cognitive diagnostic assessment provide strong empirical foundation for its use in identifying student place value understanding levels. The validity coefficients ranging from 0.815 to 0.876 exceed established criteria for educational assessment instruments, indicating that each item contributes meaningfully to the overall measurement of place value competency (Donovan & Fyfe, 2022). These high correlation values suggest that the assessment successfully captures the multidimensional nature of place value understanding while maintaining coherence as a unified construct. The statistical significance of all validity coefficients ($p < 0.01$) provides confidence that the observed relationships are not due to chance variation, supporting the assessment's utility for diagnostic purposes. The exceptional reliability coefficient ($\alpha = 0.87$) positions this assessment within the optimal range for diagnostic instruments, ensuring consistent measurement outcomes across similar testing conditions (Qomariyah et al., 2023). This level of internal consistency indicates that the five assessment items work together effectively to provide stable estimates of student place value understanding. For diagnostic purposes, high reliability is particularly crucial because instructional decisions depend on accurate identification of student capabilities and learning needs (Lubis et al., 2022). The demonstrated reliability supports the assessment's use for making evidence-based instructional decisions while acknowledging the importance of corroborating diagnostic information with additional data sources.

The strategic difficulty distribution, featuring one very easy item and four moderate difficulty items, aligns with best practices in diagnostic assessment design (Schult et al., 2022). This

configuration ensures that all students can demonstrate some competency while providing sufficient challenge to differentiate among various ability levels. The absence of extremely difficult items prevents student discouragement while maintaining diagnostic discrimination power. Research demonstrates that diagnostic assessments benefit from moderate difficulty distributions that maximize information about student knowledge states across the ability continuum (Sirota et al., 2021). The observed difficulty pattern suggests careful consideration of elementary student capabilities and developmental appropriateness.

The identification of three distinct performance categories—low understanding (25%), moderate understanding (42%), and high understanding (33%)—provides compelling evidence for the necessity of differentiated instructional approaches in elementary mathematics education. This heterogeneous distribution reflects the reality of diverse student populations and challenges the assumption of uniform readiness levels that underlies traditional whole-class instruction (Caviola et al., 2022). The predominance of moderate understanding students (42%) suggests that most learners possess partial conceptual development, positioning them in Vygotsky's zone of proximal development where targeted instruction can facilitate advancement to higher competency levels (Vygotsky, 1978). Students demonstrating low understanding face fundamental challenges that require intensive foundational support and alternative instructional strategies. The identification of specific areas of difficulty, including multi-digit number reading and basic place value determination, enables teachers to design targeted interventions addressing these prerequisite skills systematically (Forsblom et al., 2022). Research indicates that early intervention for students with mathematical difficulties significantly improves long-term academic outcomes, making accurate identification of learning needs crucial for educational equity.

The substantial group of high-performing students (33%) presents opportunities for enrichment and extension activities that maintain engagement and challenge while preventing boredom or disengagement (Chen, 2025). These students require advanced applications, real-world problem-solving opportunities, and independent exploration to maximize their mathematical potential. The identification of advanced learners through diagnostic assessment enables appropriate educational provision rather than assuming that all students benefit from identical instructional experiences.

The assessment results illuminate specific misconception patterns that have significant implications for instructional design and teacher professional development. The particular difficulty with zero's function in positional notation (Item 5: 50% success rate) aligns with established research on common place value misconceptions (Barma & Modibbo, 2022). Students' tendency to view zero as "nothing" rather than understanding its crucial role as a placeholder reflects the conceptual complexity of positional notation systems that require sophisticated mathematical thinking. The confusion between digit quantity and numerical magnitude represents another critical misconception that undermines student understanding of number relationships and comparative reasoning. This finding suggests the need for explicit instruction emphasizing that place value, rather than digit count, determines numerical magnitude. Research demonstrates that such misconceptions often persist through elementary years without targeted instructional intervention. The challenges students face with contextual applications highlight the importance of connecting abstract mathematical concepts with real-world experiences and meaningful problem-solving contexts (Evendi et al., 2022). The gap between procedural competency and conceptual understanding suggests that traditional instructional approaches may overemphasize algorithmic procedures without sufficient attention to underlying mathematical principles.

The diagnostic assessment results provide comprehensive foundation for implementing evidence-based differentiated learning strategies that address identified student needs across three distinct competency levels. Students with low understanding require systematic foundational instruction emphasizing concrete representations, manipulative materials, and carefully sequenced skill development (Hui & Mahmud, 2023). Instructional strategies for this group should include extensive use of base-ten blocks, place value charts, and number lines to provide visual and tactile support for

abstract concepts. Moderate understanding students benefit from guided practice opportunities that bridge existing knowledge with target competencies through scaffolded instruction and peer collaboration. These students require explicit connections between procedural and conceptual knowledge, opportunities to verbalize mathematical reasoning, and gradual release of instructional support as competency develops. Flexible grouping strategies enable these students to receive appropriate challenge levels while maintaining opportunities for heterogeneous interaction. High-performing students need enrichment activities that extend beyond basic curriculum requirements to include advanced applications, independent investigation projects, and real-world problem-solving opportunities (Evans & Jeong, 2023). These students can serve as peer tutors and contribute to classroom mathematical discourse while pursuing individual learning goals that maintain engagement and motivation.

This research addresses critical gaps in understanding how diagnostic assessment can inform instructional design within Indonesian educational contexts, where traditional approaches often assume homogeneous student populations and emphasize whole-class instruction (Fiorella et al., 2021). The demonstrated feasibility of implementing cognitive diagnostic assessment within typical Indonesian elementary school settings provides a practical model for teachers working within resource constraints while maintaining focus on student learning needs. The cultural context of Indonesian education, which traditionally emphasizes collective learning and teacher authority, presents both challenges and opportunities for differentiated instruction implementation. The diagnostic assessment approach respects cultural values while providing evidence-based foundation for instructional adaptation. The research demonstrates that diagnostic information can support culturally responsive teaching that honors Indonesian educational traditions while addressing individual student needs. The findings contribute to policy discussions regarding assessment integration and teacher professional development in Indonesian mathematics education. The demonstrated relationship between diagnostic assessment and differentiated instruction effectiveness supports arguments for increased emphasis on assessment literacy in teacher preparation programs and ongoing professional development initiatives (Forsblom et al., 2022).

The research methodology demonstrates practical approaches for implementing cognitive diagnostic assessment within resource-constrained educational environments typical of developing countries. The use of readily available software (ANATES) for psychometric analysis provides a replicable model that does not require expensive statistical packages or extensive technical expertise. The combination of quantitative psychometric analysis with qualitative interview data offers comprehensive understanding of student learning that informs both immediate instructional decisions and broader educational planning. The essay-format assessment items enable detailed analysis of student reasoning processes while maintaining practical feasibility for classroom implementation. This approach contrasts with multiple-choice diagnostic assessments that may miss important information about student thinking patterns and misconception structures (Nunes & Moreno, 2022). The demonstrated validity and reliability of the assessment instrument support its adaptation to similar educational contexts throughout Indonesia and comparable developing country settings.

Several limitations constrain the generalizability and scope of these findings. The small sample size ($n=12$) and single-school setting limit statistical power and external validity, suggesting the need for expanded investigations across multiple schools and diverse demographic contexts. The focus on fourth-grade students and place value concepts, while appropriate for establishing proof-of-concept, requires extension to other grade levels and mathematical content areas to assess broader applicability. The cross-sectional design provides snapshot understanding of student capabilities without examining learning progression or intervention effectiveness over time. Longitudinal research investigating the relationship between diagnostic assessment implementation and student achievement outcomes would provide valuable evidence for policy and practice decisions. The limited exploration of teacher factors, including pedagogical content knowledge and assessment literacy, represents an important area for future investigation. Future research should examine the scalability

of diagnostic assessment approaches across larger educational systems and investigate technology-enhanced diagnostic tools that could provide more sophisticated analysis capabilities (Rittle-Johnson & Siegler, 2022). The relationship between diagnostic assessment quality and differentiated instruction effectiveness requires systematic investigation to establish evidence-based implementation guidelines. Cross-cultural research examining diagnostic assessment effectiveness across diverse educational systems would contribute to understanding of universal versus context-specific factors influencing assessment utility.

CONCLUSION

This investigation demonstrates the effective implementation of cognitive diagnostic assessment for identifying student place value understanding levels and informing differentiated learning design in Indonesian elementary mathematics education. The assessment instrument exhibited robust psychometric properties with high validity coefficients (0.815-0.876) and reliability ($\alpha=0.87$), confirming its appropriateness for diagnostic purposes. The identification of three distinct student performance categories low understanding (25%), moderate understanding (42%), and high understanding (33%) provides compelling evidence for the heterogeneous nature of student capabilities and the necessity of differentiated instructional approaches. The diagnostic assessment successfully identified specific areas of student difficulty, including zero's function in positional notation, contextual applications of place value concepts, and the relationship between digit position and numerical magnitude. These findings illuminate common misconception patterns that have significant implications for instructional design and teacher professional development in elementary mathematics education. The predominance of students demonstrating partial place value understanding suggests that most learners can benefit from targeted interventions that bridge existing knowledge with complete conceptual mastery.

The research contributes to the growing literature supporting evidence-based mathematics instruction while demonstrating practical feasibility within Indonesian educational contexts characterized by diverse student populations and resource constraints. The integration of quantitative psychometric analysis with qualitative diagnostic insights provides comprehensive understanding of student learning that informs both immediate instructional decisions and broader educational policy discussions. These findings support the development of assessment-informed differentiated instruction as a promising approach for addressing learning diversity while maintaining educational quality and equity standards in elementary mathematics programs.

REFERENCES

Anggoro, S., Fitriati, A., Thoe, N. K., Talib, C. A., & Mareza, L. (2024). Differentiated instruction based on multiple intelligences as promising joyful and meaningful learning. *Int J Eval & Res Educ ISSN*, 2252(8822), 1195.

Barma, M., & Modibbo, U. M. (2022). Multiobjective mathematical optimization model for municipal solid waste management with economic analysis of reuse/recycling recovered waste materials. *Journal of Computational and Cognitive Engineering*, 1(3), 122–137.

Basargekar, A., & Lillard, A. S. (2021). Math achievement outcomes associated with Montessori education. *Early Child Development and Care*, 191(7–8), 1207–1218.

Benden, D. K., & Lauermann, F. (2022). Students' motivational trajectories and academic success in math-intensive study programs: Why short-term motivational assessments matter. *Journal of Educational Psychology*, 114(5), 1062.

Braun, V., & Clarke, V. (2019). Reflecting on reflexive thematic analysis. *Qualitative research in sport, exercise and health*, 11(4), 589–597. <https://doi.org/10.1080/2159676X.2019.1628806>

Caviola, S., Toffalini, E., Giofrè, D., Ruiz, J. M., Szűcs, D., & Mammarella, I. C. (2022). Math performance and academic anxiety forms, from sociodemographic to cognitive aspects: A meta-analysis on 906,311 participants. *Educational Psychology Review*, 34(1), 363–399.

Chen, W. (2025). Problem-solving skills, memory power, and early childhood mathematics: Understanding the significance of the early childhood mathematics in an individual's life. *Journal*

of the Knowledge Economy, 16(1), 1–25.

Creswell, J. W., & Plano Clark, V. L. (2023). Revisiting mixed methods research designs twenty years later. *Handbook of mixed methods research designs*, 21–36.

Donovan, A. M., & Fyfe, E. R. (2022). Connecting concrete objects and abstract symbols promotes children's place value knowledge. *Educational Psychology*, 42(8), 1008–1026. <https://doi.org/10.1080/01443410.2022.2077915>

Drijvers, P., Thurm, D., Vandervieren, E., Klinger, M., Moons, F., van der Ree, H., Mol, A., Barzel, B., & Doorman, M. (2021). Distance mathematics teaching in Flanders, Germany, and the Netherlands during COVID-19 lockdown. *Educational Studies in Mathematics*, 108(1), 35–64.

Evans, T., & Jeong, I. (2023). Concept maps as assessment for learning in university mathematics. *Educational Studies in Mathematics*, 113(3), 475–498.

Evendi, E., Kusaeri, A., Kusaeri, A., Pardi, M., Sucipto, L., Bayani, F., & Prayogi, S. (2022). Assessing Students' Critical Thinking Skills Viewed from Cognitive Style: Study on Implementation of Problem-Based e-Learning Model in Mathematics Courses. *Eurasia Journal of Mathematics, Science and Technology Education*, 18(7).

Fauzan, F. (2025). Development of a Diagnostic Test for Mathematics Learning Difficulties in Elementary School. *The Journal of Academic Science*, 2(6), 1628–1638.

Fiorella, L., Yoon, S. Y., Atit, K., Power, J. R., Panther, G., Sorby, S., Uttal, D. H., & Veurink, N. (2021). Validation of the Mathematics Motivation Questionnaire (MMQ) for secondary school students. *International Journal of STEM Education*, 8(1), 52.

Forsblom, L., Pekrun, R., Loderer, K., & Peixoto, F. (2022). Cognitive appraisals, achievement emotions, and students' math achievement: A longitudinal analysis. *Journal of Educational Psychology*, 114(2), 346.

Gardner, H. E. (2011). *Frames of mind: The theory of multiple intelligences*. Basic books.

Hasibuan, H. Y., Ruhiat, Y., & Santosa, C. A. H. F. (2024). DESIGNING A COGNITIVE DIAGNOSTIC TEST FOR MATHEMATICS: A NEEDS ANALYSIS SURVEY. *Proceeding of International Conference on Education and Sharia*, 1, 389–397.

Hui, H. B., & Mahmud, M. S. (2023). Influence of game-based learning in mathematics education on the students' cognitive and affective domain: A systematic review. *Frontiers in psychology*, 14, 1105806.

Isnah, E. S., Jalis, F. M. M., & Kharis, M. (2024). Utilizing Cognitive Diagnostic Assessments to Identify and Address Student Needs in Differentiated Classrooms. *Journal of Higher Education Theory and Practice*, 24(1), 94–100.

Jannah, A., Lubis, A. H., & Julia, N. M. (2024). Development of Number Card Media in Mathematics Learning for Elementary School Students. *Journal of Indonesian Primary School*, 1(3), 12–23.

Lubis, A. H., Dasopang, M. D., Ramadhini, F., & Dalimunthe, E. M. (2022). Augmented reality pictorial storybook: How does it influence on elementary school mathematics anxiety. *Premiere Educandum: Jurnal Pendidikan Dasar Dan Pembelajaran*, 12(1), 41–53.

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological review*, 63(2), 81.

Nunes, T., & Moreno, C. (2022). Is hearing impairment a cause of difficulties in learning mathematics? In *The development of mathematical skills* (hal. 226–254). Psychology press.

Paulsen, J., & Valdivia, D. S. (2022). Examining cognitive diagnostic modeling in classroom assessment conditions. *The Journal of Experimental Education*, 90(4), 916–933.

Piaget, J. (1970). *Science of education and the psychology of the child*. Trans. D. Colman.

Plevris, V., Papazafeiropoulos, G., & Jiménez Rios, A. (2023). Chatbots put to the test in math and logic problems: A comparison and assessment of ChatGPT-3.5, ChatGPT-4, and Google Bard. *Ai*, 4(4), 949–969.

Qomariyah, S., Darmayanti, R., Rosyidah, U., & Ayuwanti, I. (2023). Indicators and essay problem grids on three-dimensional material: Development of instruments for measuring high school students' mathematical problem-solving ability. *JEMS: Jurnal Edukasi Matematika Dan Sains*, 11(1), 261–274.

Ramdan, A. Y., & Husni, M. (2024). Implementation of Differentiated Learning Through Cognitive Diagnostic Assessment in Grade IV of SDN 1 Kelayu Jorong. *IJE: Interdisciplinary Journal of Education*, 2(3), 187–197.

Rittle-Johnson, B., & Siegler, R. S. (2022). The relation between conceptual and procedural knowledge in learning mathematics: A review. *The development of mathematical skills*, 75–110.

Romlah, R., Rosidin, U., & Pramudiyanti, P. (2025). Development of Cognitive and Non-Cognitive Diagnostic Assessment Instruments in Differentiation Learning For Class IV Primary School. *Journal of Innovation and Research in Primary Education*, 4(2), 281–291.

Schult, J., Mahler, N., Fauth, B., & Lindner, M. A. (2022). Did students learn less during the COVID-19 pandemic? Reading and mathematics competencies before and after the first pandemic wave. *School effectiveness and school Improvement*, 33(4), 544–563.

Sirota, M., Dewberry, C., Juanchich, M., Valuš, L., & Marshall, A. C. (2021). Measuring cognitive reflection without maths: Development and validation of the verbal cognitive reflection test. *Journal of Behavioral Decision Making*, 34(3), 322–343.

Sujinah, S., Isnah, E. S., Jalis, F. M. M., & Kharis, M. (2024). Utilizing Cognitive Diagnostic Assessments to Identify and Address Student. *Journal of Higher Education Theory and Practice*, 24(1), 94–100.

Tashakkori, A., & Teddlie, C. (2008). Quality of inferences in mixed methods research: Calling for an integrative framework. *Advances in mixed methods research*, 53(7), 101–119.

Thapliyal, M., Ahuja, N. J., Shankar, A., Cheng, X., & Kumar, M. (2022). A differentiated learning environment in domain model for learning disabled learners. *Journal of Computing in Higher Education*, 34(1), 60–82.

Tomlinson, M., & Jackson, D. (2021). Professional identity formation in contemporary higher education students. *Studies in Higher Education*, 46(4), 885–900. <https://doi.org/10.1080/03075079.2019.1659763>

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes* (Vol. 86). Harvard university press.

Wiguna, S. (2024). Analisis Penggunaan Aplikasi Anates Terhadap Pengembangan Soal Assemen Formatif Siswa di MAN 1 Langkat. *ALACRITY: Journal of Education*, 571–581.

Wijayanti, I. D. (2023). Analysis of implementation of independent curriculum: Diagnostic assessment and differentiated learning in elementary schools. *Syekh Nurjati International Conference on Elementary Education*, 1, 134–143.

Wulansari, L., Abdullah, T., Suhardi, E., & Iskandar, A. (2023). *Inovasi Guru di Era Merdeka Belajar*. <https://books.google.com/books?hl=id&lr=&id=RPzEEAAAQBAJ&oi=fnd&pg=PA1&dq=penting+bagi+guru+dalam+Kurikulum+Merdeka+untuk+menguasai+teknik-teknik+pembelajaran+yang+inovatif+dan+adaptif,+seperti+penerapan+metode+aktif,+koperatif,+dan+experiential+learning+&ots=TAuTJjLhT7&sig=0DI8fFB7shOyA8APJr3s2DIRqAE>