JOURNAL OF INNOVATION AND RESEARCH IN PRIMARY EDUCATION | ISSN 2829-775X
Vol. 3, No. 1, 2024, 11-20
https://doi.org/10.56916/jirpe.v3i1.580

Quality Index of School Education by Multiplicative Aggregation

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Keywords
National Education Policy
School education
Overall performance
Multiplicative aggregation
Critical indicators

Article History
Received yyyy-mm-dd
Accepted yyyy-mm-dd

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Abstract
Quality school education depends on a host of factors like tangible learning outcomes, efficient governance, necessary infrastructure, equitable academic opportunities, etc. For evaluation of evidence-based policy making in the education sector, the Ministry of Human Development and NITI Aayog, Govt. of India developed School Education Quality Index (SEQI) and assessed performance of States and Union Territories of India. The paper describes methodologically sound measure of overall performance in school education (OPSCI), avoiding normalization/scaling, selection of weights and covers all chosen indicators to reflect overall improvement/decline of a State/country in current year with respect to base year. The proposed index considering multiplicative aggregation of the chosen indicators has wider applications, satisfies desired properties, facilitates construction of OPSCI for India, in addition to State-wise indices. Thus, it is possible to have inter-country comparisons and inter-region comparisons. OPSCI also helps in identification of critical indicators requiring managerial attentions, plotting path of overall progress across time by a State or a country, testing statistical hypothesis regarding equality of the index for two countries/States or equality of the index for a single country/State across time using conventional *t*-tests on the logarithms of the observations. OPSCI with theoretical advantages is recommended. Future studies suggested.

INTRODUCTION

National Education Policy (NEP) 2020 adopted by the Government of India is a game-changer and paves the way to transform India into an equitable and vibrant knowledge society by providing high quality education and thereby making India a global knowledge superpower. School education is an important component of NEP 2020 where structure has been changed from traditional 10 +2 to 5+3+3+4 structure divided into three stages: Foundational (ages 3-8), Preparatory (ages 8-11), Middle (ages 11- 14),and Secondary (ages 14-18) with the aim to provide stronger foundation, fostering critical thinking, creativity and innovation, holistic development, and to break the traditional silos and encourage a learnercentric platform for comprehensive understanding of different subjects. Additional changes like changes in curriculum, teaching practices, multidisciplinary approach to teaching, competency-based assessments, more inclusive education system, etc. are likely to prepare students for the digital age and shape the future of education in India and transform India into a superpower in the knowledge economy. NEP 2020, a forward looking education policy is in line with Goal 4 (SDG 4) emphasizing on access, equity, quality, and affordability to quality education.

Quality school education depends on a host of factors including focus on tangible learning outcomes, efficient governance, provision of necessary infrastructure and ensuring equitable academic opportunities, etc. Thus, measurement of quality-related outcomes of school education is imperative for monitoring and incentivizing States and Union Territories (UTs) to improve performance of their school system.

For evaluation of evidence-based policy making in the education sector, the Ministry of Human Development and NITI Aayog, Govt. of India developed School Education Quality Index (SEQI) as a composite index (CI) in 2019 and assessed the performance of States and UTs (https://www.mhrd.gov.in/nep-new). It recognizes that school education is a subject on the Concurrent List and that State-level leadership is extremely important for improving outcomes in a cost-effective manner. The index with 30 indicators distributed over four domains (Learning Outcomes, Access Outcomes, Infrastructure & Facilities for Outcomes, and Equity Outcomes) helps States and UTs in identification of their strengths and weaknesses and undertakes requisite corrective corrections or policy interventions for improved delivery of quality education in learning levels, access, equity, infrastructure and governance processes.

However, SEQI based on a set of indicators and domains reflecting the overall effectiveness, quality and efficiency of school education system in India, suffers from methodological limitations. Normalization of raw scores by Min. – Max. transformation, arithmetic aggregation by weighted sum where sum of weights exceeds unity gives rise to problems. For example, relative performance of a State/UT due to normalization procedure may fail to have useful comparison of the State/UT across time. Mhlanga and Lall (2022**)** found that different normalization techniques resulted in different ranks to the regions. Multi-criteria decision-making (MCDM) methods like Analytic Hierarchy Process (AHP), Data envelopment analysis (DEA), Benefit of the Doubt (BoD), etc. avoid normalization, since there is no best method of normalization/scaling. For a CI using weighted sum $\sum W_i = 1$ is necessary condition to satisfy the convex property, which is not true for SEQI. Moreover, different methods are there in selecting weights to the indicators and domains. Weights could be subjective, data-driven or hybrid (Decancq and Lugo, 2009). In addition, desirable properties of the weighted sum need to be specified beforehand. Chakrabartty (2020) proposed to find weights such that $\sum_{i=1}^{n} W_i = 1$ and variance of $Y = \sum_{i=1}^{n} W_i X_i$ is minimum, if X_{ij}s, are replaced by standardized scores $Z_{ij} = \frac{X_{ij}-\overline{X_j}}{S_{X_j}}$ then the variables are equi-correlated with Y i.e. $r_{Y,Z_i} = r_{Y,Z_j} = \frac{1}{\sqrt{e^T R^{-1} e}}$ where R is the correlation matrix and $i \neq j$. Greco et al. (2019) observed that no weighting system is above criticism. Similarly, there is no ideal aggregation scheme (Arrow and Raynaud, 1986). Additive aggregation suffers from substitution problem where low value of an indicator or domain may be compensated by high value of another indicator or domain. Issues of arithmetic and multiplicative aggregations were compared by Tofallis (2014) and multiplicative aggregation was found to avoid shortcomings of the former. Geometric aggregation was preferred by Chakrabartty (2023); Segovia-González and Contreras (2023) respectively for composite index and evaluation of gender effect on educational systems of OECD countries. From 2010, Human Development Index (HDI) adopted geometric aggregation to avoid perfect substitution across dimensions (UNDP 2010). Thus, it is desirable to construct CI without considering normalization and weighted sum.

The paper finds methodologically sound measure of Index of overall performance in school education (OPSCI), avoiding normalization/scaling and selection of weights based on all chosen indicators and corresponding domains which will reflect overall improvement/decline of a State or India in current year with respect to base year or previous year.

LITERATURE SURVEY

SEQI realizes the potential of each child in India and converges efforts by the governments to evaluate education landscape which may vary with time. Summary of Index Categories and Domains is shown in Table 1.

Table 1. Summary of Index Categories and Domains

Category	Domain	Number of indicators	Total weight
1.Outcomes	1.1 Learning Outcomes	3	360
	1.2 Access Outcomes	3	100
	1.3 Infrastructure & Facilities for Outcomes	3	25
	1.4 Equity Outcomes	7	200

2.Governance Processes Aiding Outcomes	Covering student and teacher attendance, teacher availability, administrative adequacy, training, accountability and transparency	14	280
Total		30	965

For better comparisons, States and UTs have been grouped as Large States, Small States and Union Territories. Within each group, the indicator scores have been normalized by Min.-Max. transformation and weighted to generate State-wise SEQI-score depicting overall performance. States and UTs are ranked based on their overall performance in the reference year 2016-17, as well as on the change in their performance between the reference year and base year (2015-16). The rankings present status of school education across States/UTs and their relative progress over time. Domain-wise indicators and corresponding weights are given in Table 2.

Table 2. Description of domain-wise Indicators and weights

Sl. Nos.	Indicator	Weight
1.1	**Learning Outcomes**	
1.1.1	**Average score in Class 3**	
a	Language	100
b	Mathematics	100
1.1.2	**Average score in Class 5**	
a	Language	50
b	Mathematics	50
1.1.3	**Average score in Class 8**	
a	Language	30
b	Mathematics	30
1.2.1	**Adjusted Net Enrolment Ratio (NER)**	
a	Elementary level	20
b	Secondary level (Class 9 to 10)	20
1.2.2	**Transition rate**	
a	Primary to Upper-primary level	20
b	Upper-primary to Secondary level	20
1.2.3	**Percentage of identified Out-of-School Children mainstreamed in the last academic year (**Class 1 to 8)	20
1.3.1	**Computer Related Learning**	
a	Percentage of schools having Computer-Aided Learning (CAL) at Elementary level	5
b	Percentage of Secondary schools with computer lab facility	5
1.3.2	**Percentage of schools having book banks/reading rooms/libraries (Class 1 to 12)**	5
1.3.3	**Percentage of schools covered by vocational education (Class 9 to 12)**	10
1.4.1	**Absolute Difference in performance between Scheduled Caste (SC) and General Category students (**Negative valence)	
a	Language	
	Class 3	5
	Class 5	5
	Class 8	5
b	Mathematics	
	Class 3	5
	Class 5	5
	Class 8	5
1.4.2	**Absolute Difference in performance between Scheduled Tribe (ST) and General Category students (**Negative valence)	
a	Language	
	Class 3	5
	Class 5	5
	Class 8	5
b	Mathematics	
	Class 3	5
	Class 5	5
	Class 8	5
1.4.3	**Absolute difference in performance between students studying in Rural and Urban areas (**Negative valence)	
a	Language	
	Class 3	5
	Class 5	5
	Class 8	5
b	Mathematics	

	Class 3	5
	Class 5	5
	Class 8	5
1.4.4	**Absolute difference in student performance between boys and girls at Elementary level (**Negative valence**)**	
a	Language	
	Class 3	5
	Class 5	5
	Class 8	5
b	Mathematics	
	Class 3	5
	Class 5	5
	Class 8	5
1.4.5	**Absolute Difference in Transition Rate in all schools from Upper-primary to Secondary level (**Negative valence**)**	
a	SC and General Category	10
b	ST and General Category	10
c	OBC and General Category	10
d	Boys and girls	10
1.4.6	**Percentage of entitled Children With Special Needs (CWSN) receiving aids and appliances (Class 1 to 10)** *(Measured against targets set in the minutes of Project Approval Board(PAB)where the number of students receiving aids/appliances is specified)*	30
1.4.7	**Percentage of schools with toilet for girls (Class 1 to 12)**	10
2.1	**Student Attendance**	
a	Percentage of children whose unique ID is seeded in Student Data Management Information System (SDMIS)	20
b	Percentage of average daily attendance of students in SDMIS/electronic/digital database updated at least every month (Class 1 to 12) *(Note: Collected monthly data are aggregated.)*	30
2.2	**Teacher attendance**	
a	Percentage of teachers whose unique ID is seeded in any electronic database ofthe State Government/UT Administration (Class 1 to 12)	10
b	Percentage of average daily attendance of teachers recorded in the electronic attendance system *(Note: Collected monthly data are aggregated.)*	20
	Teacher adequacy	
2.3	**Percentage of single teacher schools (**Negative valence**)**	10
2.4	**Percentage of schools meeting teacher norms as per RTE Act**	
a	Percentage of Elementary schools meeting teacher norms	10
b	Percentage of Upper-primary schools meeting subject-teacher norms	10
2.5	**Percentage of Secondary schools with teachers for all core subjects (Class 9 to 10)**	10
	Administrative adequacy	
2.6	**Percentage of schools with Head- Master/Principal**	20
2.7	**Percentage of academic positions filled in State and District academic training institutions at the beginning of the given academic year** *Note: Measured against number of positions approved/sanctioned by MHRD*	
a	State Council of Education Research and Training or equivalent	5
b	District Institutes of Education and Training	10
2.8	**Percentage of teachers provided with sanctioned number of days of training in the given financial year (Class 1 to 10)**	20
2.9	**Percentage of head-masters/principals who completed School Leadership training in the given financial year (Class 1 to 12)**	15
	Accountability & transparency	
2.10	**Percentage of schools that have completed self-evaluation and made school improvement/development plans in the given financial year**	
a	Percentage of schools that have completed self-evaluation	5
b	Percentage of schools that have made school improvement/development plans *Note: Includes only those self-evaluation systems that are approved by the Department of School Education and Literacy-MHRD.*	
2.11	**Timely release of funds (**Negative valence**)** *Note: Includes funds for both Sarva Shiksha Abhiyan (SSA) and Rashtriya Madhyamik Shiksha Abhiyan (RMSA). On release of Central share of funds, the Central share is supposed to be transferred to State*	

	implementation societies within 15 days and the State share is supposed to be released to State implementation societies within 30 days.	
a	Average number of days taken by State/UT to release total Central share of funds to societies (during the previous financial year)	5
b	Average number of days taken by State to release total State share due to State societies (during the previous financial year) *Note: This indicator is not applicable for UTs. Most UTs do not contribute a State/UT share and this reduces the ability to compute and compare scores.*	5
2.12	**Number of new teachers recruited through a transparent online recruitment system as a percentage of total number of new teachers recruited in the given financial year.** *Note: The transparent recruitment system to include:* *a) annual assessment of the teacher demand – displayed online; b) written test (may or may not be online); c) online advertisement for recruitment; d) online display of marks secured by all applicants; e) online display of objective, merit-based criteria for selection; f) Transparent, online counseling for teachers.*	20
2.13	**Number of teachers transferred through a transparent online system as a percentage of total number of teachers transferred in the given year (Class 1 to 12)** *Note: The transparent online transfer system should:* *a) include a regular and annual transfer;* *b) be done on an electronic and transparent online system; c) include teacher preferences;* *d) be based on an objective transfer policy*	20
2.14	**Number of head-masters/principals recruited through a merit-based selection system as a percentage of total number of head-masters/principals recruited (in the given financial year) (Class 1 to 12)**	20

Notes:

1. Negative valence of an indicator indicates lower value of the indicator ⇒ better performance
2. Here, base year refers to 2015-16 and reference year refers to 2016-17 with the exceptions to indicators (1.1.1 to 1.1.3 and 1.4.1 to 1.4.4) for which there is no base year data and for which the reference year data is from 2017-18.
3. f a State/UT did not submit data for a required indicator, a score of 'Zero' was assigned.
4. If an indicator is Not Applicable (NA) for a State/UT, it has been excluded from the calculation, and the weight reallocated to the remaining sub-indicators (if available) or to the entire domain/category.

Observations

1. Selection of indicators appears to be exhaustive in the context of India. However, high correlations between a pair of indicators may result in multicolinearity, which is common in CIs. (Smith, 2002)
2. Some indicators are in terms of average scores (1.1.1 to 1.1.3) and indicators 1.1.3 to 1.4.4 consider absolute difference between two groups.
3. Data on the indicators are in ratio scales with fixed zero point. However, many indicators are in percentages. Strictly speaking, addition or subtraction or averaging of figures in percentages are not meaningful when the denominators are different and not multiple of the other. For example, 80% (8 out of 10) + 50 %(6 out of 12) is 63.64% (14 out of 22) and not 65% = $\frac{80\%+50\%}{2}$. Human Poverty Index (HPI) (UNDP, 2007) used cubic root and 4-th root of average of percentage figures to get respectively HPI-1 and HPI-2.
4. Meaningful addition or subtraction of $X \pm Y = Z$ requires similar distribution of X and Y and also knowledge of distribution of Z. Interpretation and further operations on X ± Y are problematic when X and Y follow two unknown different distributions (Arvidsson, 2019).
5. Sum of the weights $\neq 1$

Normalization

The selected indicators in SEQI are in different units and differ in score-ranges and distributions. Raw scores of indicators are normalized for making them unit free and have a common score-range. SEQI uses normalization of indicator-wise data by Min – Max function like $Y = \frac{X - X_{Min}}{X_{Max} - X_{Min}}$ for indicator with positive valence and $Y = \frac{X_{Max} - X}{X_{Max} - X_{Min}}$ for indicator with negative valence where$0 \leq Y \leq 1$. It depends heavily on X_{Max}andX_{Min} which could be unreliable outliers. Major disadvantages of such transformation are as follows:

a) *Y* -score of one state is relative to performance of others. Performance of a third state can influence relative ranking of two states (Kasparian and Rolland, 2012).
b) Decrease in performances of the worst performing state/UT may increase *Y*-value of a state even if raw score for the state remains unchanged.
c) Actual value of a State/UT (in %) – minimum value in a group of States/UTs (in %) could be problematic. Thus, Min – Max transformation may not handle well the outliers.
d) -If X_{Min} is changed, rankings of the States and UTs may get changed, difference in variance for the changes is not eliminated (OECD, 2008). Gain in *Y* due to unit increase in *X* varies for different ranges of values of *X*.
e) The zero point of raw data gets shifted by such transformation.
f) If X_{Min} is changed, ranking and relative valuations of the States and UTs may get changed due to change in marginal rates of substitution (Seth and Villar, 2017) and difference in variance for the changes is not eliminated (OECD, 2008).
g) Gain in *Y* due to unit increase in *X* varies for different ranges of values of *X*.
h) Min – Max transformation fails to satisfy Translation Invariance property and consistency in aggregation which are considered desirable (Chakravarty, 2003)

Overall performance as weighted sum

Different weights assigned to the indicators appear to be subjective without considering evidence based data on relative importance. Overall performance of the *i*-th state for the *t*-th year ($SEQI_{it}$) is computed as $SEQI_{it} = \frac{\sum w_i . Y_i}{\sum w_i}$

Thus, ranking of the States and UTs may be done with respect to $SEQI_{it}$ or with respect to $[SEQI_{it} - SEQI_{i(t-1)}]$ or $[SEQI_{it} - SEQI_{i\,t_0}]$ where t_0 denotes the base year.

However, to ensure that measurement of $SEQI_{it}$ lie in a convex set, it is necessary to select weights so that sum of weights is equal to one. In such scenario, $SEQI_{it}$ could be defined as $SEQI_{it} = \sum_{i=1}^{n} W_i X_i$ for $0 < W_i < 1$ and $\sum_{i=1}^{n} W_i = 1$ (Example: Environmental Performance Index (2016) or $SEQI_{it} == \frac{1}{n} \sum_{i=1}^{n} W_i X_i$ in line with Wellbeing Index (ElSarawy, 2016). Here, the ratio $\frac{W_1}{W_2}$ indicates the amount to be sacrificed from the indicator-2 to gain an extra unit of indicator -1. Changing weights to indicators affect CI of the State being evaluated (Saisana et al. 2005) and can manipulate ranking of States (Grupp and Schubert, 2010). There are various ways to select weights to the indicators even satisfying $\sum_{i=1}^{n} W_i = 1$ and no weighting system is above criticism (Greco, et al. 2019)

Limitations of SEQI

1. Index for the entire country was not evaluated.
2. The analysis of incremental performance excluded data on learning outcomes since learning outcomes data in the reference year and the base year was not comparable due to changes in test items, coverage and reporting scales.
3. Subjective weights based on consultations with sector experts
4. Does not find relative importance of domains/dimensions and indicators in $SEQI_{it}$
5. Fails to identify critical domains or indicators showing poor for necessary corrective policy action.
6. Fails to test whether $[SEQI_{it} - SEQI_{i(t-1)}]$ is statistically significant or not

Fails to draw path of progress of $SEQI_{it}$ registered by the *i*-th State across time.

METHODS

Pre-processing of data

Convert each indicator with negative valence to positive valence by taking reciprocal of the indicator and ensure that for each indicator higher value ⇒ better performance.

Construction of the Index

Let value of the *i*-th indicator at *t*-th time period of a State/UT be denoted by $X_{it} > 0$ for i =1,2,….*n*. For the base period, denote X_{it} by X_{i0}. The unit free ratio $\frac{X_{it}}{X_{i0}}$ indicates improvement or decline registered by the State/UT at *t*-th time period over the base period with respect to the *i*-th indicator. Index of overall performance in school education (OPSCI) for the current time-period may be defined as the Geometric mean of *n*-indicators as

$$OPSCI_{c0} = \sqrt[n]{\frac{X_{1c}, X_{2c}, \dots\dots, X_{nc}}{X_{10}\, X_{20} \dots\dots X_{n0}}}$$ or avoiding the *n*-th root,

$$OPSCI_{c0} = \frac{X_{1c}, X_{2c}, \dots\dots, X_{nc}}{X_{10}\, X_{20} \dots\dots X_{n0}} \qquad (1)$$

Similarly, OPSCI for India aggregating all the States and UTs is given by

$$OPSCI_{India_{c0}} = \prod_{j=1}^{k} OPSCI_{j-th\ State/UT_{c0}} \qquad (2)$$

where *k* denotes total number of States/UTs on which data were collected.

$OPSCI_{c0} > 1$ ⟹ Overall improvement from the base year.

$\frac{OPSCI_{i_t}}{OPSCCI_{i_{(t-1)}}} > 1$ indicates the progress made by the *i*-th State/UT in *t*-th year over (*t*-1)-th year. For two successive years, critical indicators are those for which $\frac{X_{it}}{X_{i(t-1)}} < 1$ and can be focused for policy purpose to decide appropriate action. Each of (1) and (2) can be applied for all types of indicators in different score-ranges including those in percentages and quantifies overall improvement or decline in the current year from the base year. Replacing the base period vector by the vector for the previous year will give improvement in OPSCI on year-to-year basis.

It may be noted that the proposed index is an example of multiplicative aggregation of the indicators. Taking logarithm on both sides of (1) we get

$$\log(OPSCI_{c0}) = \sum_{i=1}^{n} \log\left(\frac{X_{ic}}{X_{i0}}\right) \qquad (3)$$

Log-transformed geometric standard deviations (log GSD) is given by

log (GSD) = usual SD of logY_1, log Y_2,, ………, logY_n where $Y_i = \frac{X_{ic}}{X_{i0}}$

Equation (3) converts the multiplicative aggregation to additive aggregation. Such conversion helps to test $H_0: OPSCI_{ti} = OPSCI_{tj}$ for two different countries *i* and *j* or H_0: $OPSCI_{ti} = OPSCI_{(t-1)i}$ for the *i*-th country can be tested using conventional *t*-tests on the logarithms of the observations (Alf and Grossberg, 1979; Koh et al. 2018).

RESULTS AND DISCUSSION

Result

The index OPSCI by (1) is monotonically increasing continuous function, avoids scaling and weights, produces no bias for large and small States, since effects of outliers is not there. The index satisfies the following desired properties:

1. Significant reduction of substitutability among the indicators or domains.
2. Time-reversal test since $OPSCI_{t0}.OPSCI_{0t} = 1$
3. Here, $OPSCI_{20} = OSCI_{21}.OPSCI_{10}$. Thus, chain indices can be formed.

4. Chain indices can be used to plot path of overall progress across time by a State or a country.

Discussion

Avoiding scaling and selection of weights, the paper provides a simple method of construction of Index of overall performance in school education at *t*-th year ($OPSCI_t$) by multiplicative aggregation facilitating aggregation of the indicators at *t*-th year to get $OPSCI_t$ for the *j*-th State/UT at *t*-th year $(\mathfrak{D}_{i_t})$ and further aggregation of $\mathfrak{D}_{i_t}s$ to get country score $(OPSCI_{India,t})$. $OPSCI_{India,t}$ reflects position of India at the *t*-th year by a continuous monotonically increasing variable as an absolute measure satisfying desired properties like meaningful aggregation, Time-reversal test, formation of chain indices, etc. and offering significant benefits. $OPSCI_t$ can better handle the outliers and produces no bias for developed or underdeveloped State/UT. The index helps to compute relative importance of the domains and identify the critical domains or indicators requiring attention of the policy makers to initiate necessary remedial action. Using longitudinal date, the index computes extent of overall progress/decline of a State or the country as a whole and to draw progress path of *OPSCI*. The States may also be compared with respect to such progress paths.

The proposed method contributes to improve aggregation of SEQI indicators avoiding major limitations of the existing methods of weighted sum of normalized scores of the indicators, offers the following advantages:

1. Significant reduction of level of substitutability
2. Aggregated scores of all indicators = Aggregated scores of domains; aggregated scores of all domains = Aggregated scores of the State/UT and aggregated scores of all States/UTs = Aggregated scores of the country.
3. Requires no selection of weights or scaling/normalizing indicator scores.
4. Facilitates construction of overall performance in school education (*OPSCI*) for a country, in addition to State-wise indices. Thus, it is possible to have inter-country comparisons and inter-region comparisons for a given year or similar comparisons across years.
5. Also facilitates construction of domain-wise indices considering indicators relevant to that domain.
6. Easy to find relative importance of each indicator or domain.
7. Identification of critical indicators requiring managerial attentions.
8. Plotting of path of overall progress across time by a State/UT or a country using the Chain indices
9. Statistical hypothesis regarding equality of $OPSCI_t$ for two States for the *t*-th year or equality of $OPSCI_{Indiat_1}$ and $OPSCI_{Indiat_2}$ or the *j*-th State at two different years can be tested using conventional *t*-tests on the logarithms of the index.
10. The index can be used for better ranking, classification of States/UTs and facilitates computation of mean and variance of a sample of States.

Each of $OPSCI_{c0}$ by equation (1) and $OPSCI_{India_{c0}}$ by equation (2) can be multiplied by 100 to have parity with common interpretation of an index.

CONCLUSION

The existing multi-dimensional measure of SEQI has several disadvantages. The proposed *OPSCI* depicting overall improvement/decline of a country or a State in the current year with respect to base year or previous year has significant theoretical advantages and is recommended. Policy makers and researchers can take advantages of the multiplicative aggregation of the proposed index. Future studies may be undertaken to investigate finer points of the proposed index including robustness and sound method of combining growth curves of *n*-States/UTs to estimate assumption free hazard function and validations.

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