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Application of theories on thinking to build the theory of mathematical thinking in teaching high school mathematics

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Abstract
The paper explores the development of a theory on mathematical thinking in the teaching of Mathematics at the high school level (HSL). The first part of the paper presents characteristics of thinking, including problem-solving, direct and indirect aspects, abstraction, generalization, and its close relationship with language. The process of thinking formation is described through various stages, where direct and indirect thinking holds a significant position. The following section classifies types of thinking such as logical thinking, imaginative thinking, and argumentative thinking. Additionally, it categorizes thinking into different levels of independence. Thinking is classified based on the object, the field of perception, and the thinking method. The crucial part of the paper focuses on mathematical thinking, describing it as a creative process applying mathematical methods, concepts, and procedures to solve problems. In conclusion, the paper constructs a theory of mathematical thinking in teaching Mathematics at HSL, aiming to develop and enhance mathematical capabilities in students by grasping fundamental knowledge and skills. It encourages core mathematical abilities such as thinking, modeling and problem-solving.

INTRODUCTION

Mathematics is not only a fundamental subject in the education system but also a crucial foundation for the holistic development of students. To achieve this goal, understanding mathematical thinking and its application in teaching is essential. This paper focuses on exploring how we can apply

theories of thinking to construct an innovative and effective theory of mathematical thinking in teaching high school mathematics.

The paper delves into various aspects of the theory of mathematical thinking, from basic concepts to key characteristics and their application in the teaching process. By combining general thinking theories with specific objectives of high school mathematics, the paper discusses the theory of mathematical thinking in the teaching of high school mathematics in Vietnam. It incorporates specific teaching methods, from problem-based learning to creative practice and multidimensional assessment, aimed at developing mathematical thinking for students.

The objective of the paper is not only to provide a deep insight into mathematical thinking but also to offer teachers and learners tools and strategies to enhance mathematical ability and creative thinking in the learning process. In doing so, we hope to contribute to the improvement of the quality of teaching and learning mathematics in high schools.

RESULTS AND DISCUSSION

Theories of Thinking

Concept of Thinking

According to Phan Dung (2013), thinking is a positive reflection of reality, associated with solving various problems; it is the highest product of the specially organized material – the human brain. The result of thinking is thoughts that solve problems.

According to Tran Khanh Duc (2012), thinking is the mental, intellectual, conscious activities of human beings with the material origin being the brain. Thinking is not only the process of perception, reflection (directly or indirectly) but also the process of action (action thinking or transformation) to solve practical problems and adapt to the objective reality (nature, society, and the individual).

According to Edgar Morin (2009), thinking is the highest method of organizing the activities of the mind, which, through language, establishes a perception of reality and its worldview.

In summary, thinking is an intellectual activity that impacts the brain, reflecting objective perceptions of phenomena in daily human society life to solve problems and relationships.

Characteristics of Thinking

Problematic nature of thinking: Problems are states of phenomena, events, situations, or circumstances that are new, arising in theory as well as in mental and spiritual life and practical activities. To solve problems, humans seek solutions through thinking (reasoning) with new ways, new problem-solving methods.

Direct and indirect nature of thinking: Human thinking directly perceives the world through senses (such as eyes, nose, ears, legs, hands, etc.) and indirectly through language – a signaling system (concepts, formulas, laws, etc.) and practical experiences to analyze, compare, generalize, abstract, model, simulate, and predict in the past, present, and future.

Abstract and general nature of thinking: Thinking has an abstract nature, using the mind to retain essential elements and eliminate unnecessary elements of perceived phenomena. Generalization is the process of categorizing different objects into groups, types, based on similar attributes or relationships, with certain laws.

Thinking and language: Thinking and language (writing and speaking) have an intimate relationship. Language is the means and tool to express the results of thinking.

In conclusion, through these characteristics, human thinking is a subjective activity that arises when a "problem" appears. Individuals must have sufficient awareness and will before the task of identifying and solving it (the task). At the same time, thinking directly or indirectly reflects the general essence of the phenomena, turning them into object groups through language.

Thinking Operations

According to Nguyen Quang Uan (1999), the research results in psychology describe thinking operations as follows:

Analysis: The process of using the mind to divide the perceived object into parts, different components, from which to identify the attributes, characteristics of the perceived object or determine the parts of a whole through comparison, classification, making the whole clear.

Synthesis: The process of using the mind to unite, arrange, or combine parts, elements, or attributes of the perceived object that have been separated through analysis into a coherent whole. In thinking, synthesis is considered an operation with a creative mark. When saying someone has a "synthetic mind," it is similar to saying someone has a "creative mind."

Comparison - similarity: A thinking operation aimed at "identifying the similarities and differences between the phenomena of reality." Through comparison, one can find the essential and non-essential similarities and differences between objects. It can also discover the essential and non-essential signs of them.

Abstraction: Abstraction is the process of using the mind to remove aspects, attributes, relationships, and secondary relationships, and only retain the typical, essential factors of the perceived object.

Generalization: The process of using the mind to integrate many different objects into one group, one type, based on common attributes, relationships, or the essence of the object or phenomenon. The result of generalization is to produce a common characteristic of a series of objects of the same type or create new perceptions in the form of concepts, laws, rules.

Classification of Thinking

According to Nguyen Canh Toan, Nguyen Van Le, and Chau An (2005), thinking is classified into the following types:

Basic, common types of thinking: logical thinking (based on middle and syllogistic logic), dialectical thinking, and imaginal thinking.

In terms of independence, thinking is divided into four levels: dependent thinking, independent thinking, critical thinking (dialectical), and creative thinking.

Based on the characteristics of the object for thinking, thinking is divided into two types: abstract thinking and concrete thinking.

According to Tran Khanh Duc (2021), there are various ways to classify thinking, such as:

According to the history of formation and the level of development of thinking, there are three basic levels: visual-action thinking, visual-imaginal thinking, and abstract thinking. Another classification is based on the hierarchy of cognitive levels in education by B. Bloom and adjustments proposed by Anderson (2001) from the level of knowing/remembering... to evaluating and creating.

According to the classification of thinking areas: philosophical, political thinking; social thinking; economic thinking; educational thinking; scientific thinking; technical thinking and technology...

According to the field of perception: philosophical thinking, scientific thinking, artistic thinking.

According to pairs of opposites: Thinking can also be classified according to pairs of opposites or cause-and-effect relationships, such as theoretical - practical thinking; logical - illogical thinking; concrete - abstract thinking; low-level - high-level thinking...

According to the thinking method: Creative thinking, critical thinking, logical thinking, systematic thinking.

In conclusion, the various classifications of thinking demonstrate the methods, modes, objects of thinking in the practical social life that are diverse, bringing benefits to each individual in society.

Bloom's Taxonomy of Thinking Levels:

Each thinking level represents a different level of complexity in the learner's cognitive process. The levels are arranged from low to high, with lower levels requiring less thinking than higher levels.

Bloom's Taxonomy of thinking levels is widely used in education, especially in developing educational objectives and testing assessments. The use of this taxonomy helps teachers clearly identify the learning objectives they want students to achieve, while also constructing questions and exercises suitable for each thinking level. The first taxonomy of thinking levels was developed by Benjamin S. Bloom (1956), commonly known as Bloom's Taxonomy, consisting of the following six levels: (1) Knowledge; (2) Comprehension; (3) Application; (4) Analysis; (5) Synthesis; (6) Evaluation.

Recognizing that the scale above is not entirely complete, in the mid-1990s, Lorin Anderson, a student of Benjamin Bloom, along with some colleagues, proposed adjustments as follows (Pohl, 2001): (1) Remembering; (2) Understanding; (3) Applying; (4) Analyzing; (5) Evaluating; (6) Creating.

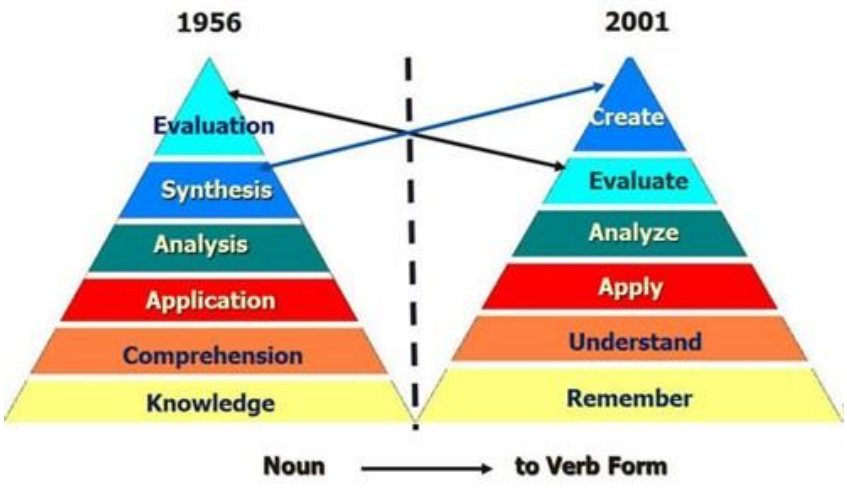

Figure 1. Bloom's Cognitive Domain (Pohl, M. (2001))

Compared to Bloom's taxonomy of cognitive levels, Anderson's taxonomy undergoes some notable changes:

The lowest cognitive level is Remember instead of Know. This indicates that the ability to recall information is an essential skill that needs to be developed in learners.

The Synthesis level is omitted, and Create is added to the highest level. This change emphasizes the importance of developing creative abilities in learners.

Verbs are replaced with nouns. This clarification enhances the understanding of cognitive activities that learners need to perform at each level.

Mathematical Thinking

According to Henderson, Hichtner, and colleagues (2002), mathematical thinking involves using mathematical techniques, concepts, and procedures to solve problems directly or indirectly.

According to Liu (2003), mathematical thinking is a combination of processes such as prediction, generalization, interpretation, description, abstraction, sampling, formal and informal reasoning, verification, and complex processes based on definitions and explanations already made about mathematical thinking.

According to Mason, Burton & Stacey (2010), mathematical thinking is a dynamic process that helps us understand complex structures more easily by combining our ideas.

Author Alan H. Schoenfeld (2014) focuses on defining and explaining the importance of mathematical thinking. The author defines mathematical thinking as "the process of searching, problem-solving, reasoning, analyzing, and interpreting information in a logical and accurate manner. Mathematical thinking not only involves calculating or solving mathematical equations and problems but also relates to using mathematical skills to solve problems in daily life."

According to Bukova (2006), mathematical thinking is considered as obtaining new knowledge or concepts by abstracting, estimating, generalizing, formulating and testing hypotheses, insightful thinking, proving, and determining using mathematical knowledge and concepts that individuals have learned before.

In conclusion, mathematical thinking involves using all methods, tools, techniques, concepts, whether directly or indirectly, throughout the process of solving mathematical problems.

Developing the Theory of Mathematical Thinking in Teaching High School Mathematics

High School Mathematics Subject

The content of the high school mathematics curriculum analyzed and used in this paper follows the Vietnamese Ministry of Education and Training (MOET) (2018) guidelines for mathematics education.

1. The high school mathematics subject aims to help students achieve the following main objectives:

Contribute to the formation and development of mathematical abilities with the requirements of being able to: pose and answer questions in reasoning, problem-solving; use reasoning, induction, and deduction methods to understand different approaches in problem-solving; establish mathematical models to describe situations, thereby proposing solutions to mathematical problems within the established models; perform and present problem-solving solutions and evaluate the implemented solutions, reflecting on the value of the solutions, generalizing for similar problems; use tools and mathematical learning means in studying, exploring, and solving mathematical problems.

Acquire fundamental mathematical knowledge and skills, especially in algebra and some elements of analysis, geometry and measurement, statistics, and probability.

Contribute to students having a relatively general understanding of professions related to mathematics and its value; lay the foundation for career orientation after high school; have sufficient minimum capacity to self-learn issues related to mathematics throughout life.

2. Achievable Requirements

Mathematics contributes to the formation and development of students' mathematical abilities (the most concentrated expression of computational ability) consisting of the following core components: mathematical thinking and reasoning ability; mathematical modeling ability; mathematical problem-solving ability; mathematical communication ability; ability to use tools and means of learning mathematics.

Theory of Mathematical Thinking

The theory of mathematical thinking is a crucial aspect in the teaching of mathematics in high schools. It emphasizes not only the transmission of mathematical knowledge but also focuses on the development of students' thinking abilities, helping them grasp and solve mathematical problems in an innovative and effective manner.

1. Characteristics of the Theory of Mathematical Thinking:

The theory of mathematical thinking typically revolves around several key characteristics aimed at encouraging the holistic development of mathematical abilities:

a) Creative thinking

The theory of mathematical thinking aims to encourage creativity in the problem-solving process. Students are not only encouraged to apply learned knowledge but are also challenged to find new solutions, different perspectives for each problem.

b) Reflective thinking

The ability to evaluate and reflect is an essential part of mathematical thinking. Students are encouraged not only to understand how to solve a problem but also to assess the rationality of methods and results, helping them develop analytical and logical reasoning skills.

c) Systematic thinking

The theory of mathematical thinking sets requirements for the ability to build and understand mathematical systems. Students learn how to connect concepts, rules, and methods to create a comprehensive view of the problem, helping them apply knowledge to new situations.

2. Teaching Methods in Mathematics Based on the Theory of Mathematical Thinking:

a) Problem-based learning:

This method places the problem at the center of the learning process. Instead of transmitting knowledge, teachers create real-life situations or complex problems, encouraging students to use mathematical thinking to solve them.

b) Creative practice:

Students are encouraged to participate in creative activities such as group discussions, experiments, and individual projects. This helps them apply mathematical thinking to real-life situations and develop teamwork skills.

c) Multidimensional assessment:

The assessment system needs to reflect the holistic development of students, not only through memorization but also through application, reflection, and creativity in problem-solving.

3. Application of the Theory of Mathematical Thinking in Teaching High School Mathematics:

The theory of mathematical thinking is one of the crucial theories in mathematics education, guiding the teaching of high school mathematics. It suggests that mathematical thinking is the process of perceiving mathematical objects, including activities such as analysis, synthesis, abstraction, generalization, inference, proof, and problem-solving. Bloom's taxonomy is a classification tool for learning objectives according to cognitive levels, from low to high, including:

a) Remembering: Ability to recall learned information, such as symbols, formulas, definitions, mathematical theorems, etc.
b) Understanding: Ability to explain and interpret learned mathematical information.
c) Applying: Ability to apply mathematical knowledge and skills to solve problems.
d) Analyzing: Ability to break down mathematical information into smaller components and identify relationships between those components.
e) Evaluating: Ability to make judgments, evaluate mathematical information based on appropriate criteria.
f) Creating: Ability to establish new mathematical information based on existing mathematical knowledge.

Applying the theory of mathematical thinking and Bloom's taxonomy in teaching high school mathematics:

Applying the theory of mathematical thinking and Bloom's taxonomy in teaching high school mathematics can help teachers develop students' mathematical thinking abilities at higher levels. Specifically, teachers can implement the following measures:

Promote Memory and Reinforce Basic Knowledge:

Through memorization and recall of symbols, formulas, definitions, mathematical theorems, and specific steps to solve a particular problem.

Connect Mathematics to Reality:

Problems in the high school mathematics curriculum need to be linked to real-life situations, helping students apply mathematical knowledge to solve practical problems. Connecting mathematics to reality will help students understand the significance of mathematics, stimulating thinking and interest in learning mathematics.

Create Opportunities for Independent Thinking and Problem-Solving:

Teachers need to create opportunities for students to think independently and solve problems, avoiding the passive transmission of knowledge. Allowing students to think independently and solve problems will help them develop creative thinking skills, solve mathematical problems quickly and logically.

Classify Students According to Thinking Abilities:

Teachers need to classify students according to thinking abilities to have suitable remediation measures. Classifying students will help teachers orient content, teaching methods suitable for each group of students, thereby helping students develop mathematical thinking abilities effectively. Understanding the "Theory of Mathematical Thinking" and how it relates to teaching high school mathematics, we can analyze the following important points:

Enhancing Cognitive Abilities:

The theory of mathematical thinking helps students develop and enhance their cognitive abilities. This includes logical thinking, analysis, synthesis, comparison, and generalization.

Mathematical Modeling:

Mathematical thinking is often accompanied by the ability to model, meaning the ability to graphically represent a mathematical problem into models. This supports a deeper understanding of mathematical problems and how they relate to reality.

Mathematical Problem-Solving:

The theory of mathematical thinking provides a foundation for efficient mathematical problem-solving. Students are encouraged to use mathematical techniques and concepts to solve complex problems.

Promoting Creativity:

Mathematical thinking is not only about rules and logic but also addresses the ability to be creative in problem-solving. This can foster creativity and creative thinking in students.

Mathematical Communication:

The theory of mathematical thinking is also related to the ability to communicate mathematically, meaning the ability to present ideas and explain the problem-solving process clearly and logically.

Application of Mathematics in Daily Life:

Mathematical thinking exists not only in academic environments but also has applications in everyday life. Linking the theory of mathematical thinking with the teaching of high school mathematics can help students see the practical value of mathematical knowledge.

By applying the theory of mathematical thinking to teaching, teachers can create a positive learning environment, encouraging curiosity and deep understanding of mathematics. This will help students not only learn about mathematics but also develop critical thinking skills essential for success in various areas of life. The theory of mathematical thinking establishes a bridge between the content

of knowledge and students' application abilities in high school mathematics. Applying this theory to the teaching process not only helps students gain a deep understanding of mathematics but also develops essential thinking skills for success in both academic and future life.

CONCLUSION

In this paper, we have examined the concepts of thinking and mathematical thinking, particularly in the context of high school education. Theories of thinking have been discussed, encompassing the notions, characteristics, stages, and operations of thinking. This helps us gain a deeper understanding of the thinking process and how it influences how humans solve problems and interact with the surrounding world.

We delved into the theory of mathematical thinking, a crucial aspect when applying thinking to the subject. This theory goes beyond the use of mathematical tools and techniques; it involves the process of searching, problem-solving, reasoning, and interpreting information in a logical and precise manner. Mathematical thinking not only aids students in understanding the structure of mathematics but also develops fundamental and essential mathematical abilities.

In general, applying the theory of thinking to the teaching of mathematics at the high school level plays a crucial role in developing mathematical abilities and exploring students' interest in the subject. The integration of the theory of thinking and the educational objectives of high school mathematics can help students not only grasp knowledge but also develop creative thinking skills and problem-solving flexibility.

In summary, researching and applying the theory of mathematical thinking in teaching can positively contribute to the comprehensive development of students, from basic thinking skills to the ability to solve complex problems in mathematics specifically and real-life situations in general.

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